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2.1 INTRODUCTION

2.2 BEHAVIOR FOR TORSION

2.3 DESIGN MODEL

2.4 CALCULATION FOR TORSION

2.5 DETAILING OF REINFORCEMENT

Introduction / Introducere

BASIC LOADS IN A RC CROSS SECTION

AXIAL FORCES

NORMAL STRESSES
$\sigma\ [N/mm^2]$

CENTRIC COMPRESSION/TENSION
$N\ [kN]$
BENDING MOMENT
$M\ [kNm]$

TANGENTIAL FORCES

TANGENTIAL STRESSES
$\tau\ [N/mm^2]$

SHEAR FORCE
$V\ [kN]$
TORSION
$T\ [kNm]$

Introduction / Introducere

COMPLEX LOADS IN A RC CROSS SECTION

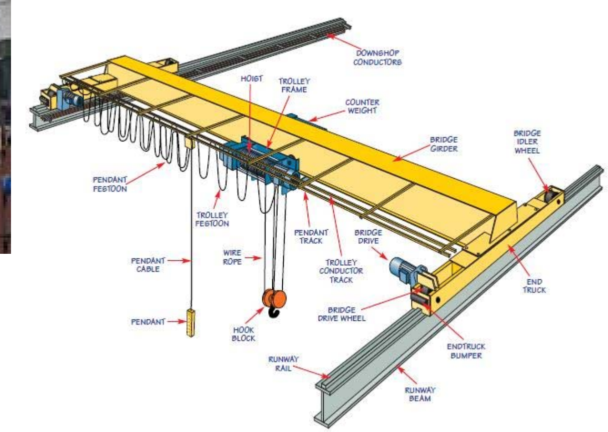
AXIAL FORCES	TANGENTIAL FORCES
COMPRESSION/TENSION WITH ECCENTRICITY $M\ [kNm] \pm N\ [kN]$	SHEAR COMBINED WITH TORSION $V\ [kN] + T\ [kNm]$

Causes of torsion a) structural continuity
b) space configuration of structures

Introduction / Introducere

Examples when torsion arise

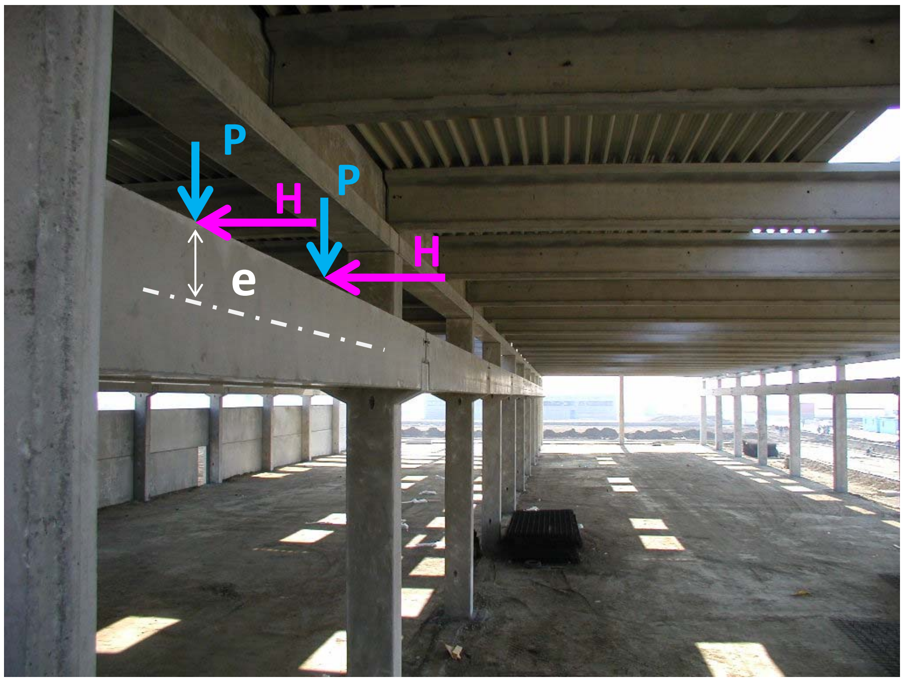
→ RUNWAY GIRDER FOR CRANE BRIDGE



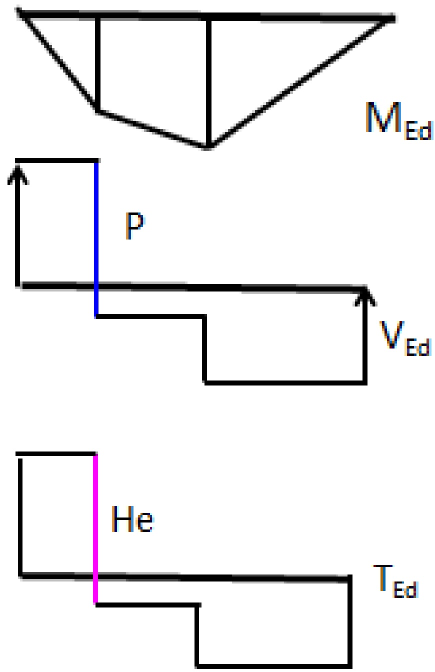
Introduction / Introducere

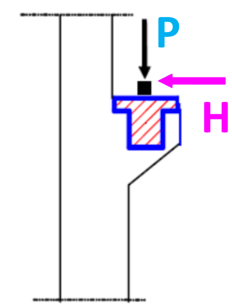
Examples when torsion arise

→ RUNWAY GIRDER FOR CRANE BRIDGE



Applied torque = H x e
(cuplu)

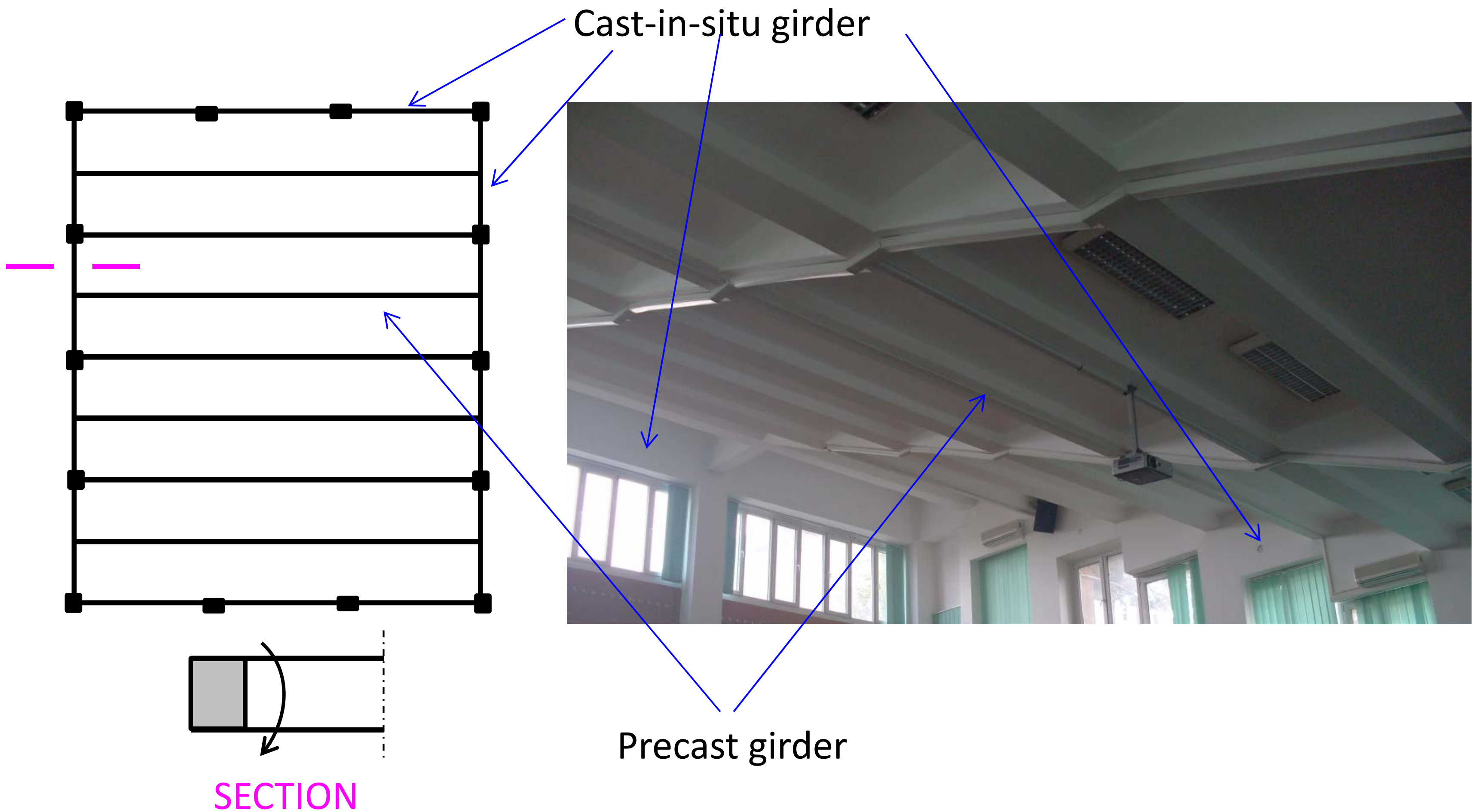




Introduction / Introducere

Examples when torsion arise

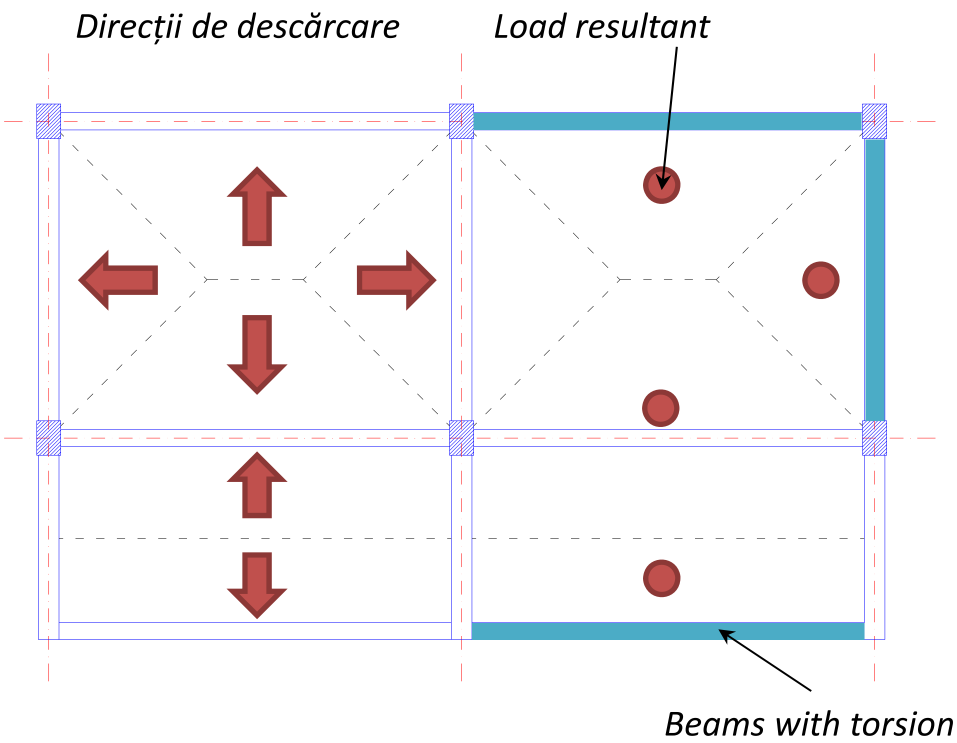
→ CAST-IN-PLACE EDGE BEAMS (FLOOR OF THE AUDITORIUM)



Introduction / Introducere

Examples when torsion arise

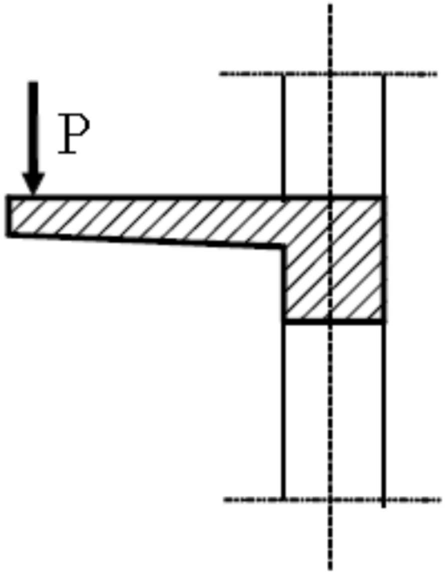
→ BEAMS WITH SLAB IN CANTILEVER



Introduction / Introducere

Examples when torsion arise

→ BEAMS WITH SLAB IN CANTILEVER

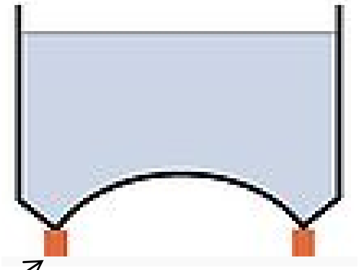


Introduction / Introducere

Examples when torsion arise

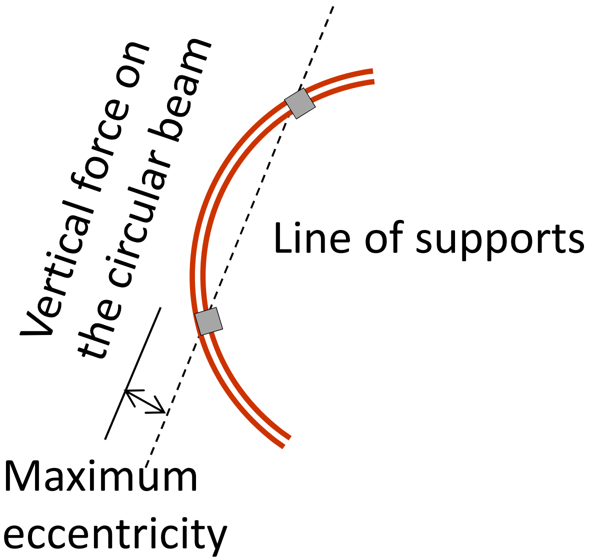
→ CURVED BEAMS → INTZE WATER TANK

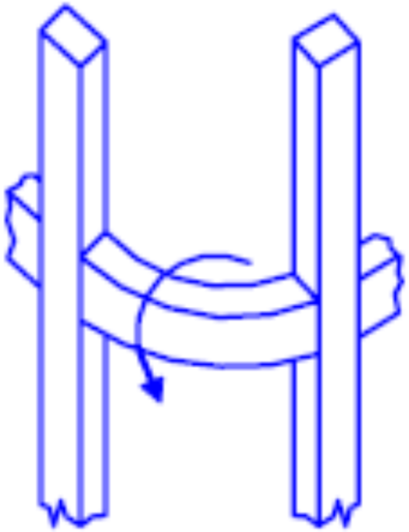
(1843 – 1902)



Only vertical forces are transmitted to the tower

Circular beam

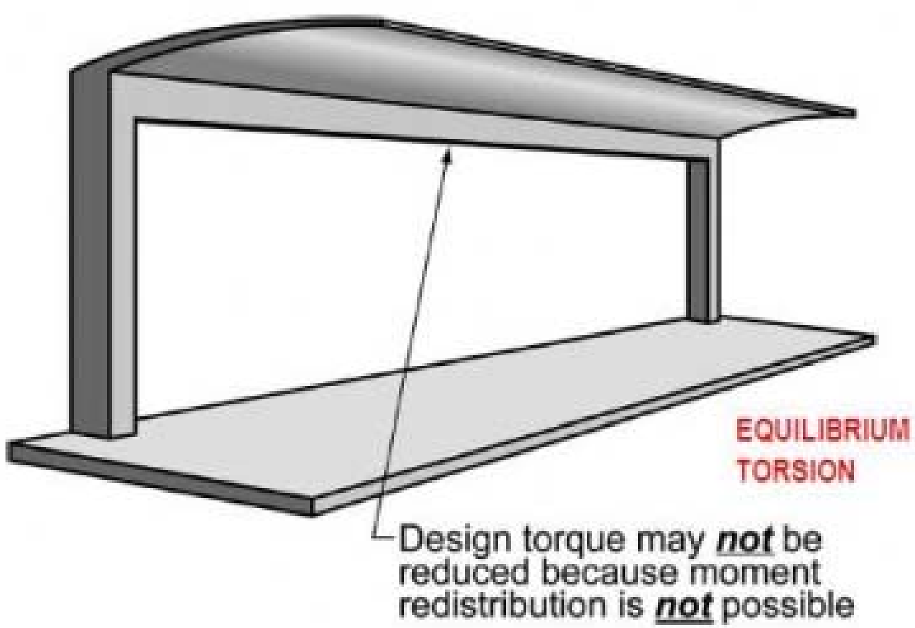


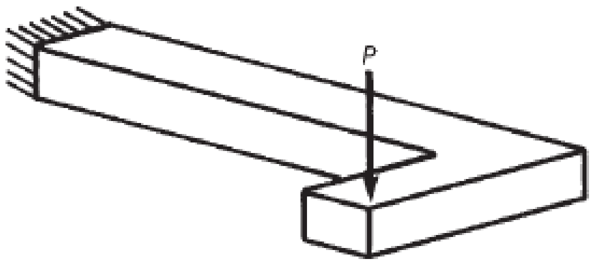


EQUILIBRIUM (PRIMARY) TORSION

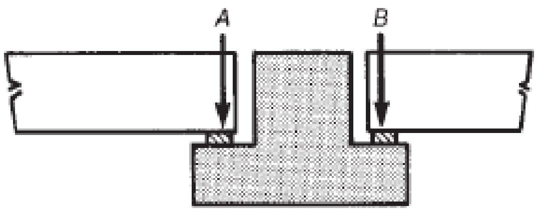
- static equilibrium of a structure depends on the torsional resistance
- full torsional design shall be made

Canopy





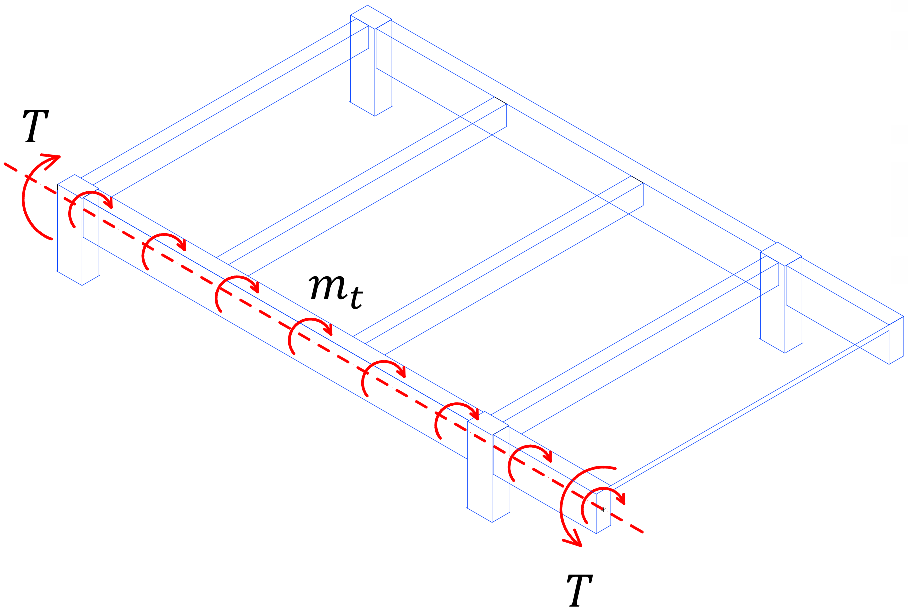
(a) Cantilever beam with eccentrically applied load.

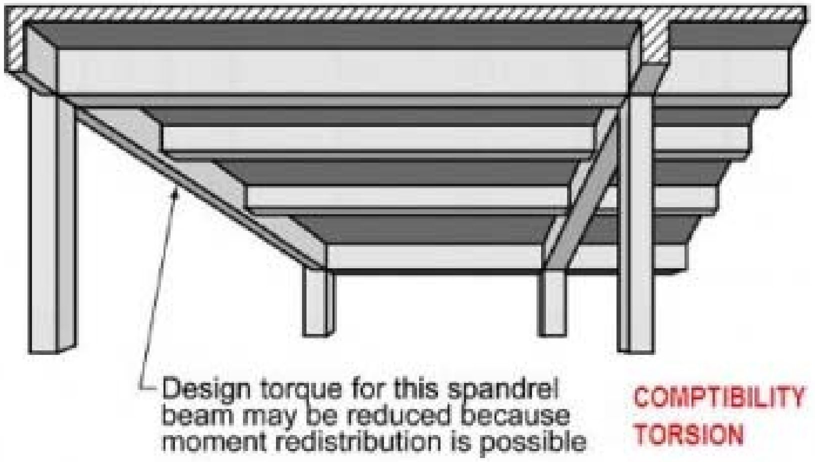


(b) Section through a beam supporting precast floor slabs.

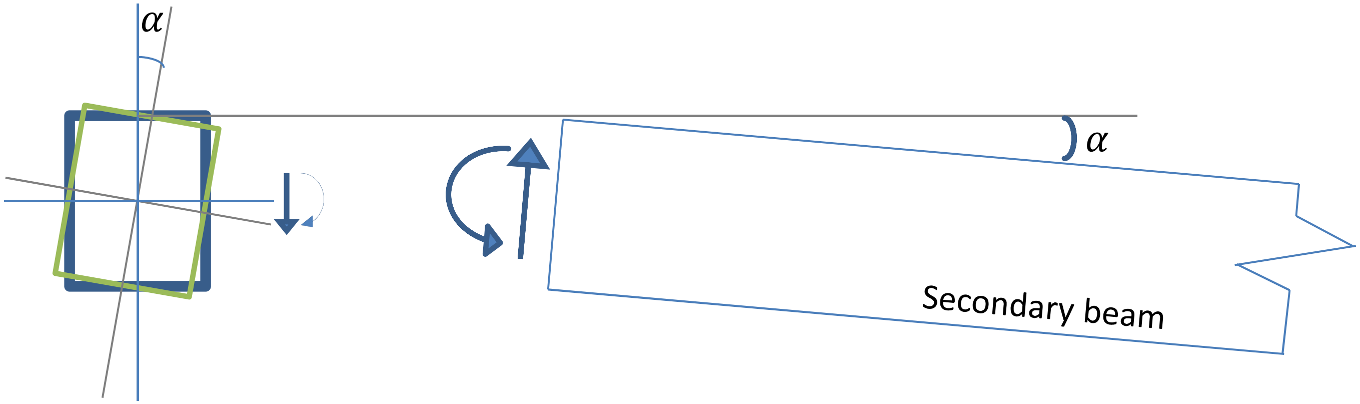
COMPATIBILITY (SECONDARY) TORSION

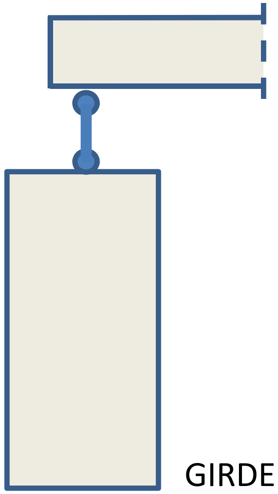
- torsion arises from consideration of compatibility
- the structure is not dependent on the torsional resistance
- it will normally be unnecessary to consider torsion at the ultimate limit state
- a minimum reinforcement should be provided





COMPATIBILITY (SECONDARY) TORSION





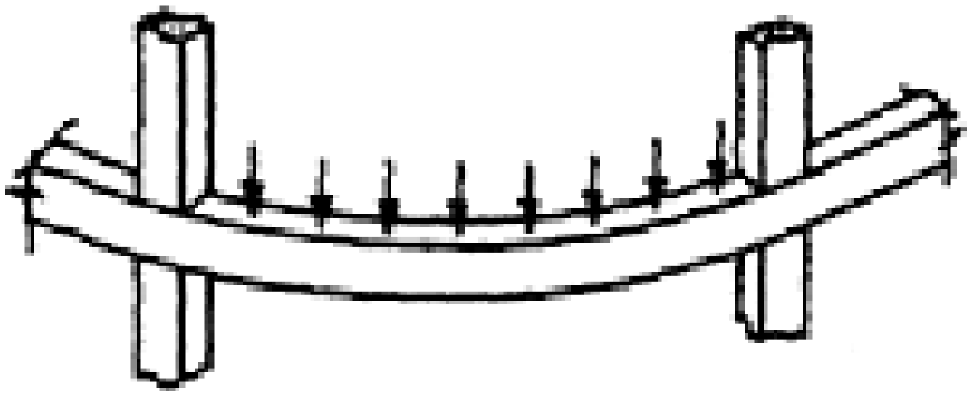
Torsional stiffness of the girder is very low compared with the secondary beam → Redistribution of stresses

Secondary beam:

- Torsional stiffness could be neglected
- Design in ULS is not necessary
- Supplementary longitudinal reinforcement will be placed in zone with negative moment

Introduction / Introducere

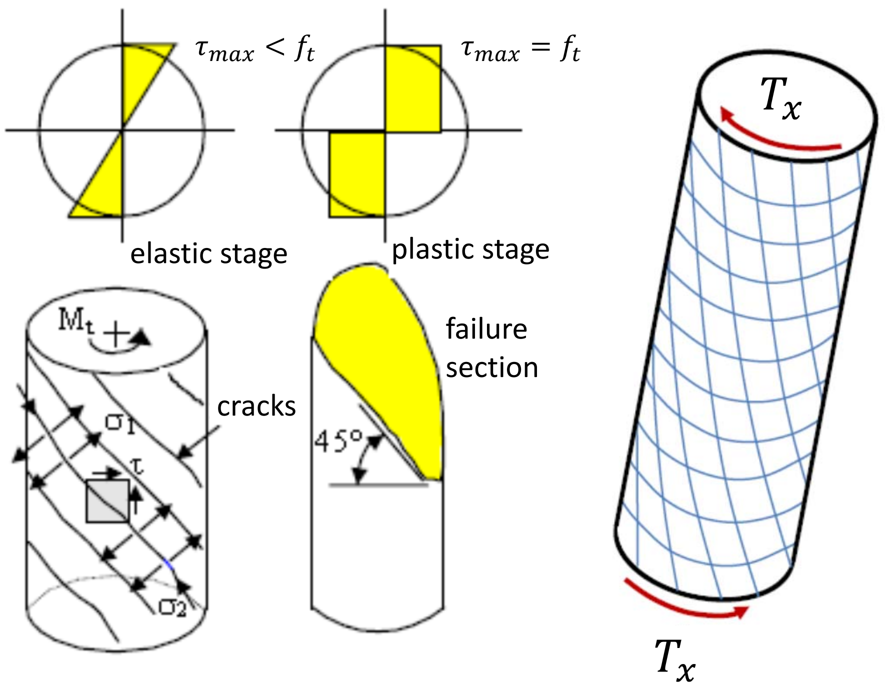
COMPATIBILITY & EQUILIBRIUM TORSION



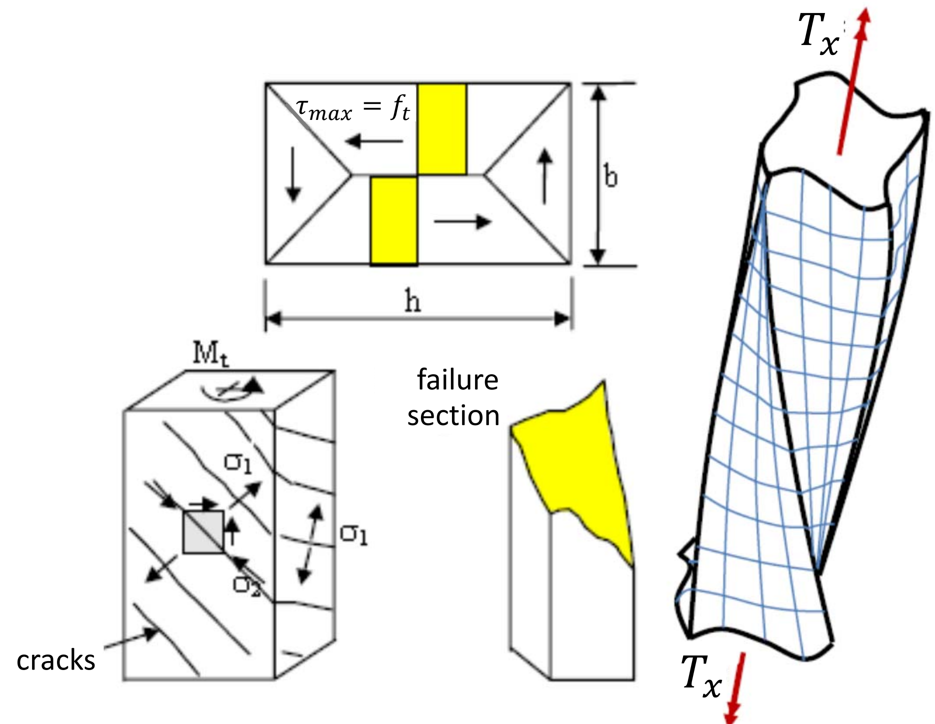
Introduction / Introducere

PLAIN CONCRETE TORSION

Failure of plain concrete for torsion



concrete cylinder



concrete prism

Cross sections before and after torsion remains plain
→ the principle of plain cross sections is valid

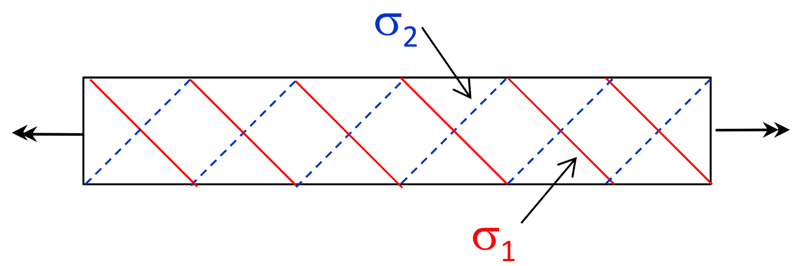
Due to torsion the cross sections do not remain plain
→ the principle of plain cross sections does not applicable, it is not valid

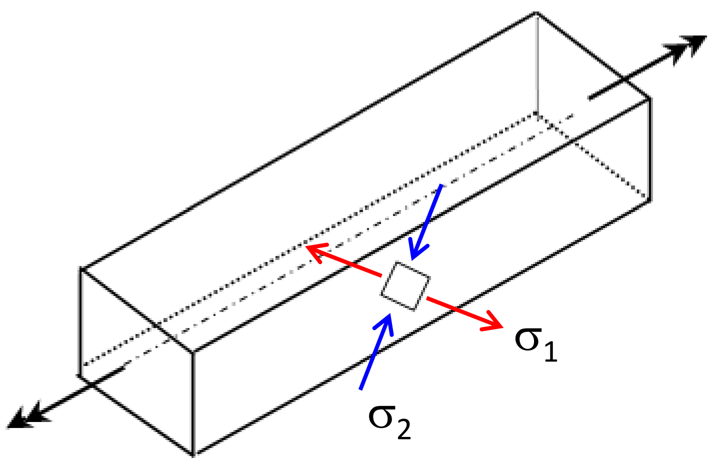
(Prof. Clipii T.)

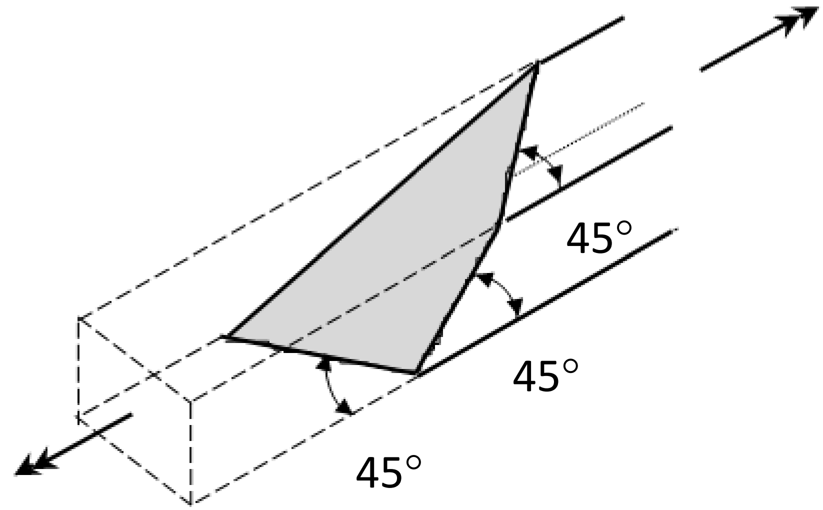
PLAIN CONCRETE TORSION

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \rightarrow \text{at the neutral axis level } \sigma_1 = \tau_0$$

Trajectories of principal stresses





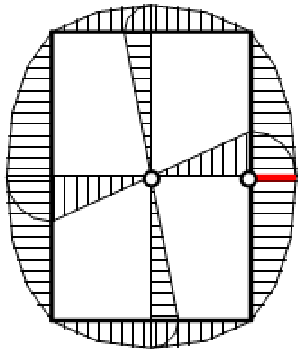


SKEWED SECTION

(Prof. Clipii T.)

PLAIN CONCRETE TORSION

If it is accepted that in the moment of failure the concrete is fully plasticized:



$$\sigma_1 = \tau_{max} = \frac{T}{W_t} \rightarrow f_{ct}$$

and

$$W_t = \frac{1}{6} b^2 h \left(3 - \frac{b}{h}\right)$$

where

W_t - section modulus for torsion

b - the smallest dimension of the section

h - the greatest dimension of the section

(for b and h:) whichever the section orientation is

(Prof. Clipii T.)

Behavior for torsion / Comportarea la torsiune

FAILURE IN TORSION

a) PLAIN CONCRETE

$$T_{cracking} = T_R$$

b) REINFORCED CONCRETE

→ longitudinal reinforcements in the corners of the cross section + closed stirrups

$$T_{cracking} < T_R$$

Failure of elements subjected to $M_{Ed} + V_{Ed} + T_{Ed}$

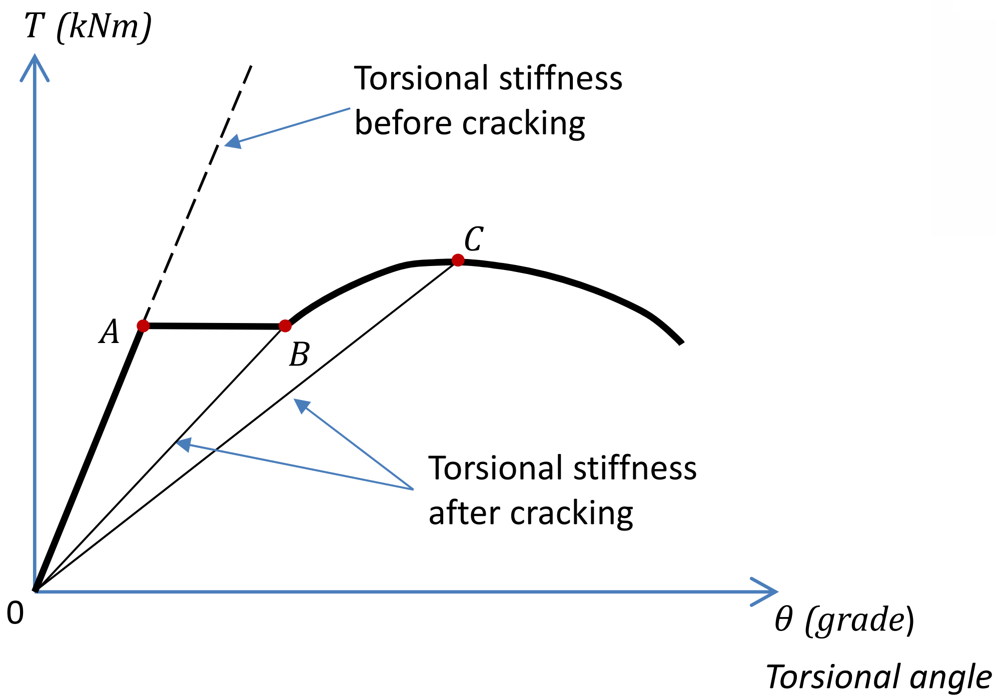
→ yielding of reinforcements in skewed section (longitudinal and/or transversal) followed by crushing of compression concrete

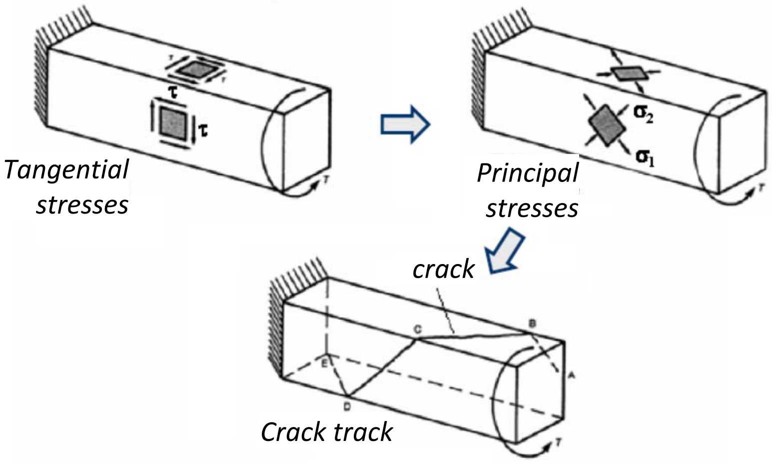
OR

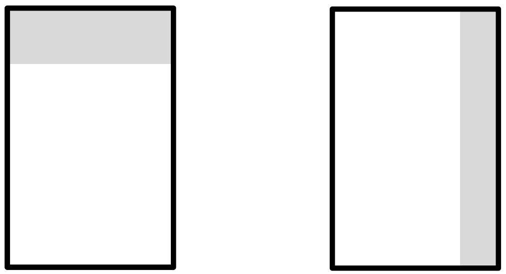
→ crushing of compression concrete (for over-reinforced elements) ↔ brittle failure → must be avoided

Behavior for torsion / Comportarea la torsiune

$T - \theta$ relation





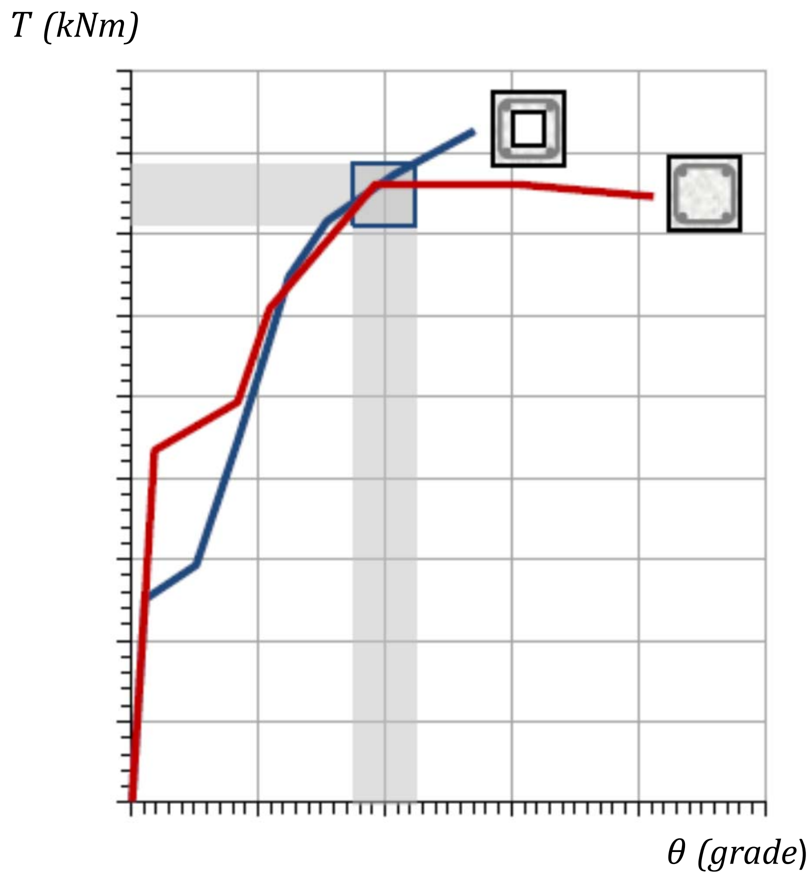


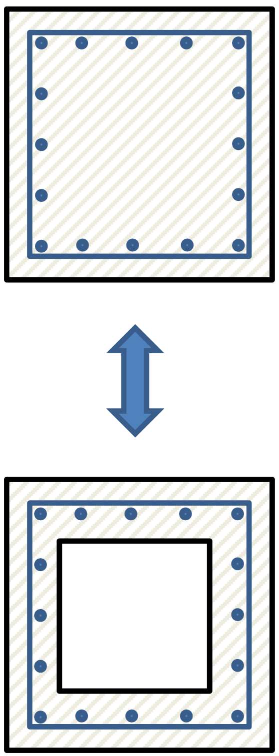
Depending on the correlation between M_{Ed}, V_{Ed}, T_{Ed} the compressed concrete position could be changed.

Behavior for torsion / Comportarea la torsiune

Experimental tests on RC elements were demonstrated that:

→ Differences in capacity for torsion of rectangular solid and tubular cross sections are not important, thus the contribution of the core concrete could be neglected for torsion calculations.

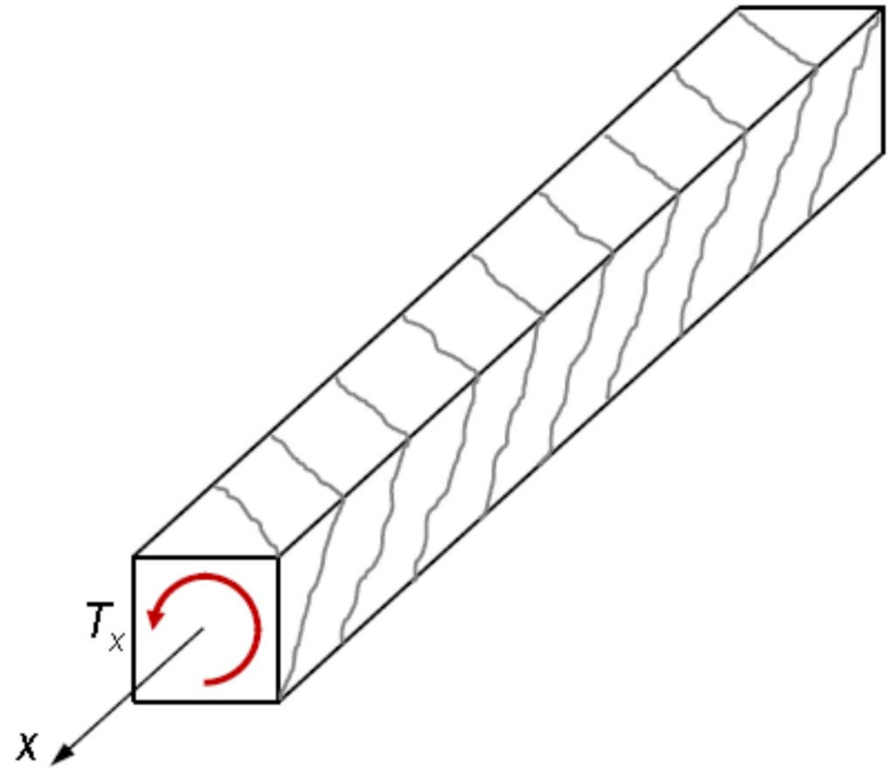




Behavior for torsion / Comportarea la torsiune

Experimental tests on RC elements were demonstrated that:

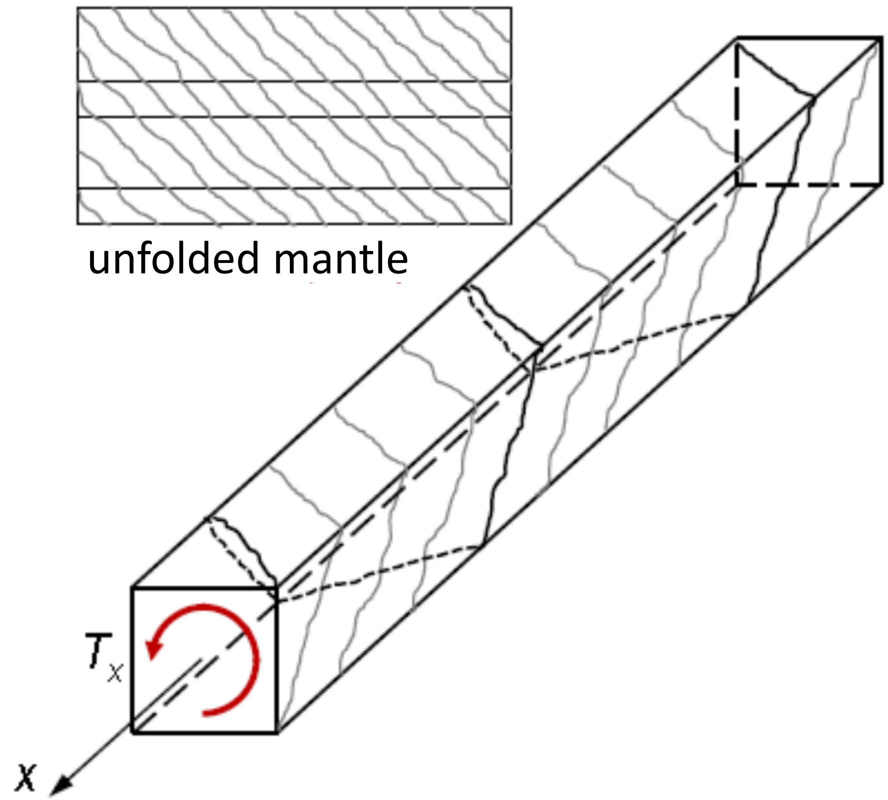
→ Crack pattern of a RC beam subjected to torsion is very similar with an element subjected to shear



(Prof. Kovács I, DE)

Behavior for torsion / Comportarea la torsiune

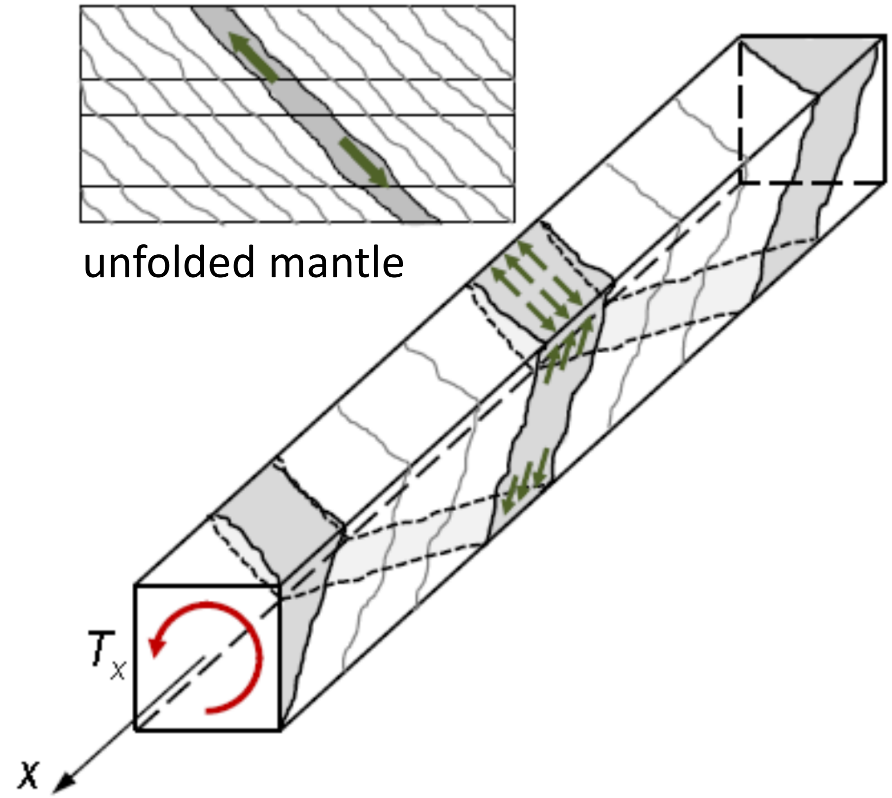
Experimental tests on RC elements were demonstrated that:
→ Cracks resulting from torsion are forming coherent and continues crack pattern peripherally to the beam



(Prof. Kovács I, DE)

Behavior for torsion / Comportarea la torsiune

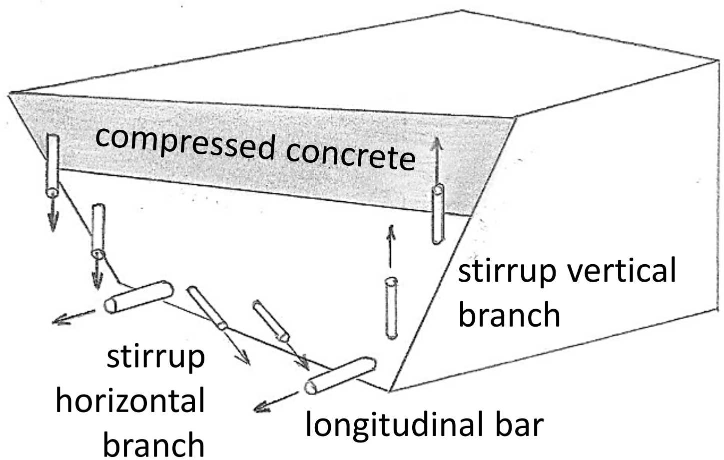
Experimental tests on RC elements were demonstrated that:
→ Concrete zones between torsional cracks could be considered compressed concrete bars



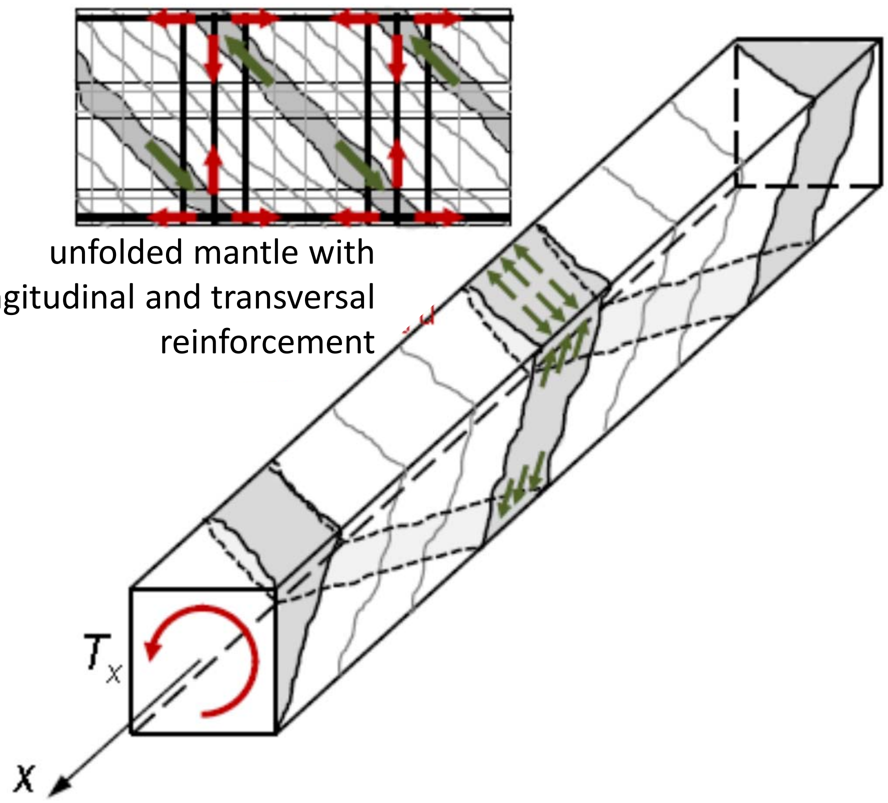
(Prof. Kovács I, DE)

Behavior for torsion / Comportarea la torsiune

→ Tensile stresses perpendicular to torsional cracks are taken by the longitudinal and transversal reinforcements from the cross section



Limit equilibrium model



(Prof. Kovács I, DE)

Behavior for torsion / Comportarea la torsiune

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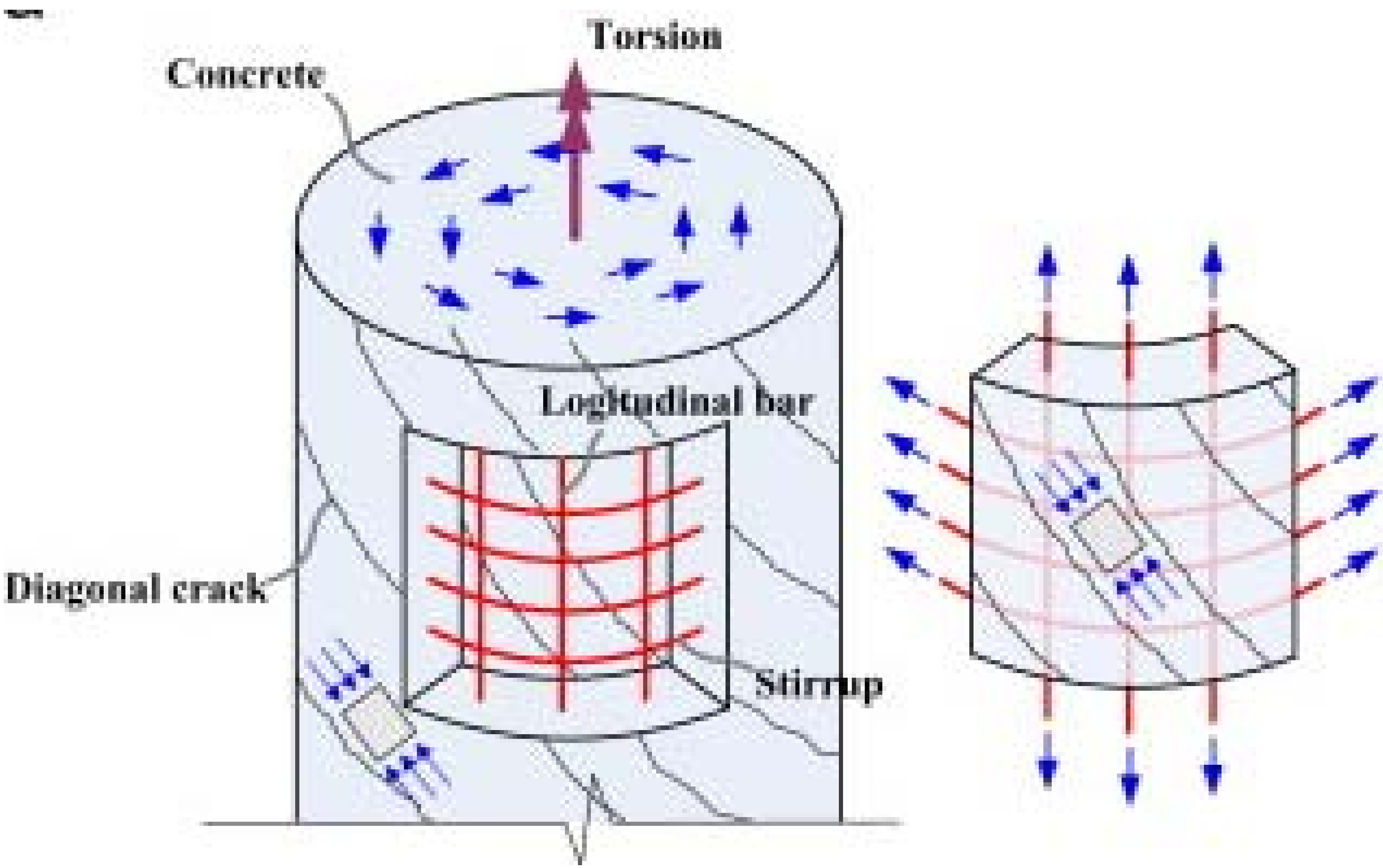

Limit equilibrium model

Slide 26

TNG1 collins and mitchel - Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams, PCI J 1980
Tamas Nagy Gyorgy, 28-Feb-17

Behavior for torsion / Comportarea la torsiune

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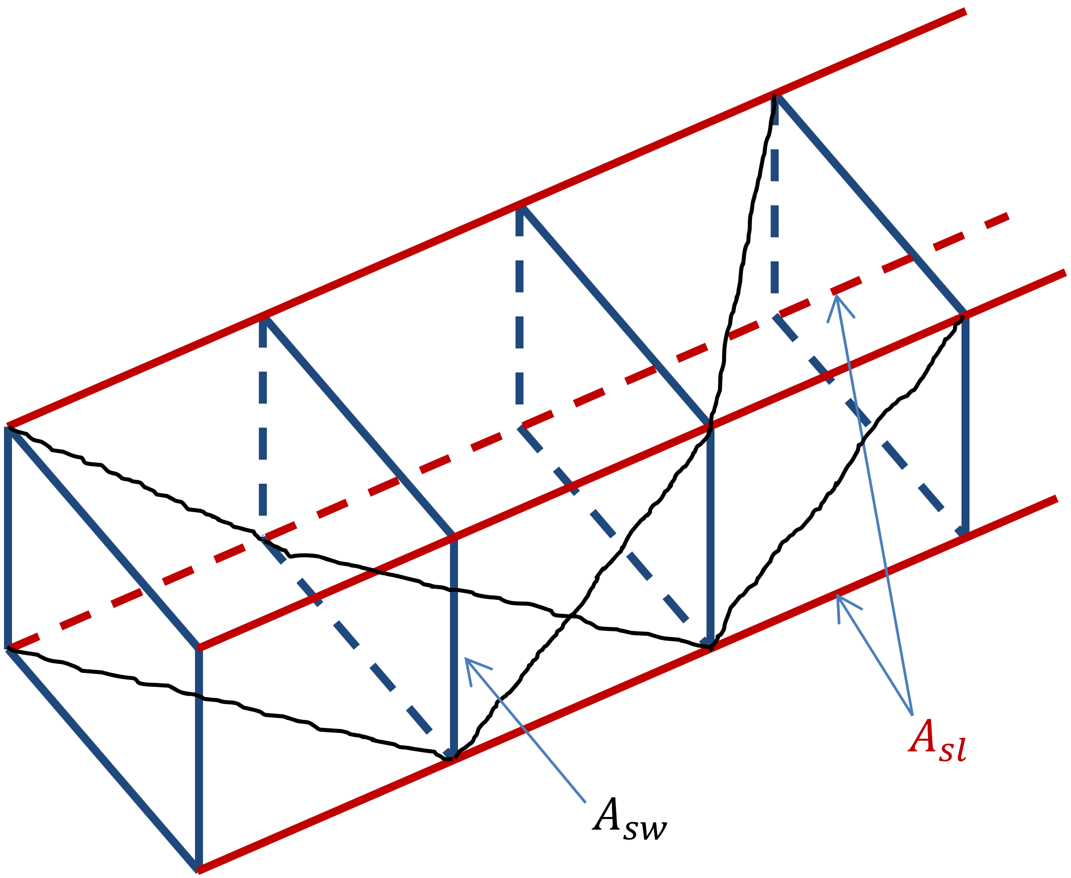
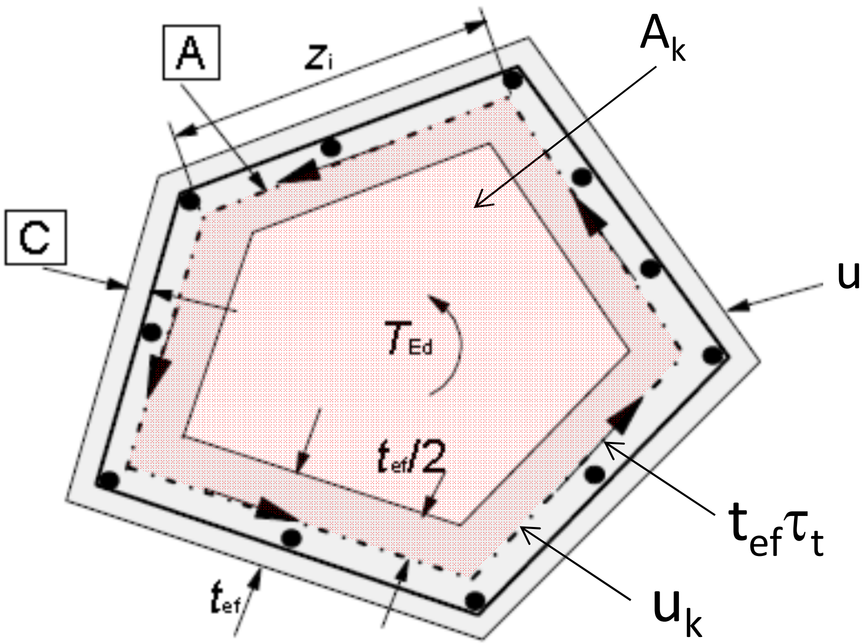


Space truss model

Design model / Modelul de calcul

The torsional resistance of a section may be calculated on the basis of a thin-walled closed section.



A – center-line, enclosing area A_k

A_k – area enclosed by the center-line, including inner hollow areas

u_k – perimeter of area A_k

u – outer perimeter

z_i - side length of wall i

τ_t – torsional shear stress

c – concrete cover

t_{ef} – effective wall thickness

Equilibrium is satisfied by a closed shear flow $t_{ef}\tau_t$

Design model / Modelul de calcul

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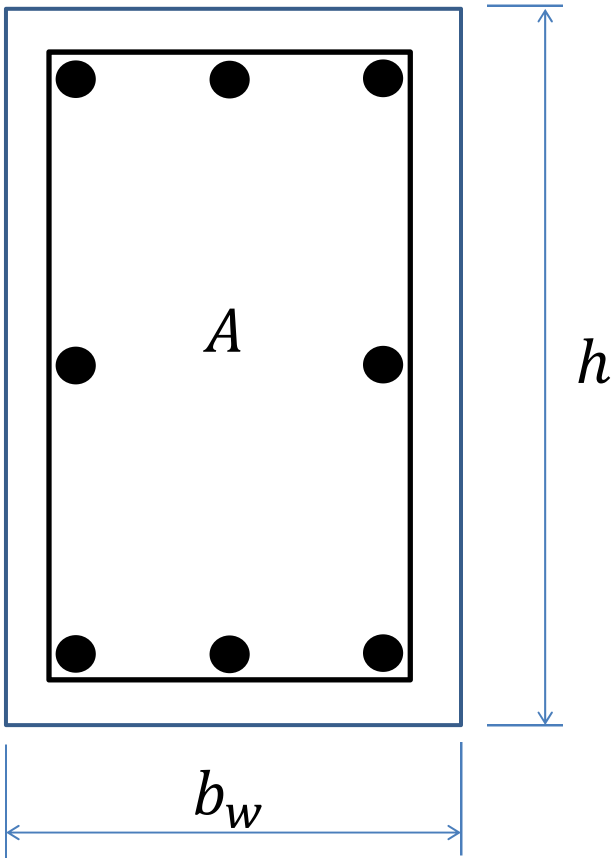

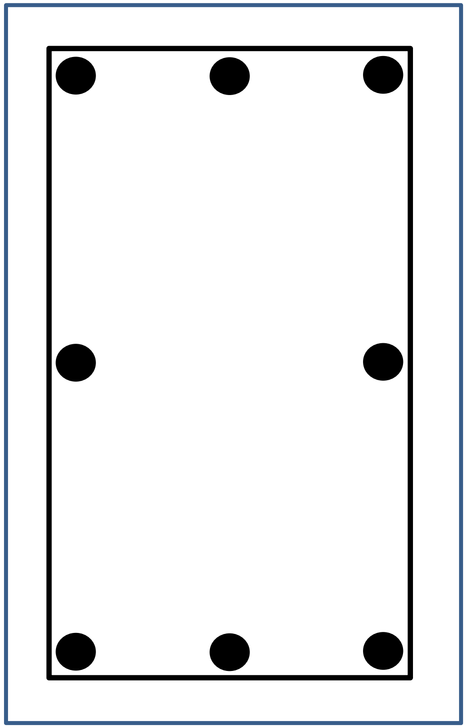

$$A = b_w h$$
$$u = 2(b_w + h)$$

Design model / Modelul de calcul

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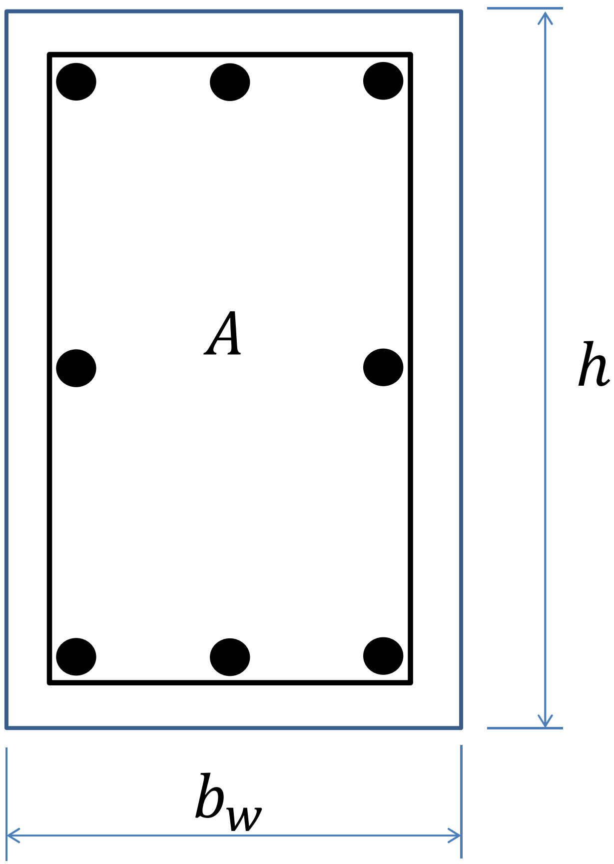

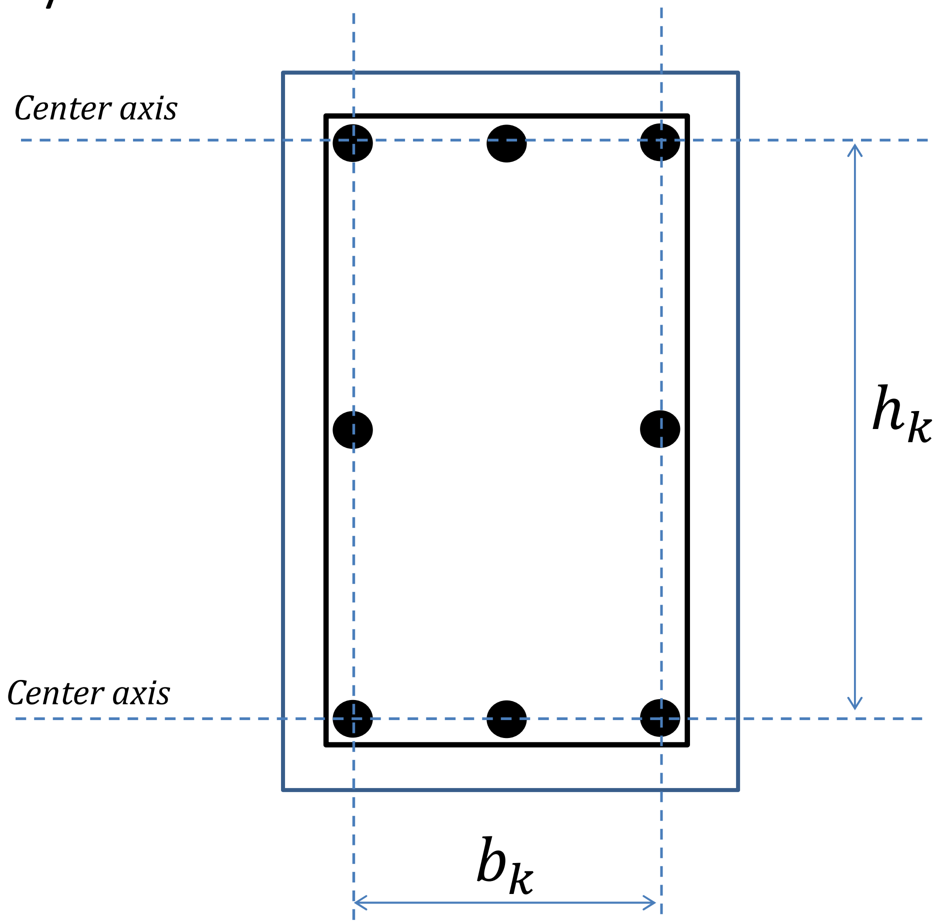

$$A = b_w h$$
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$$A_k = b_k h_k$$
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Design model / Modelul de calcul

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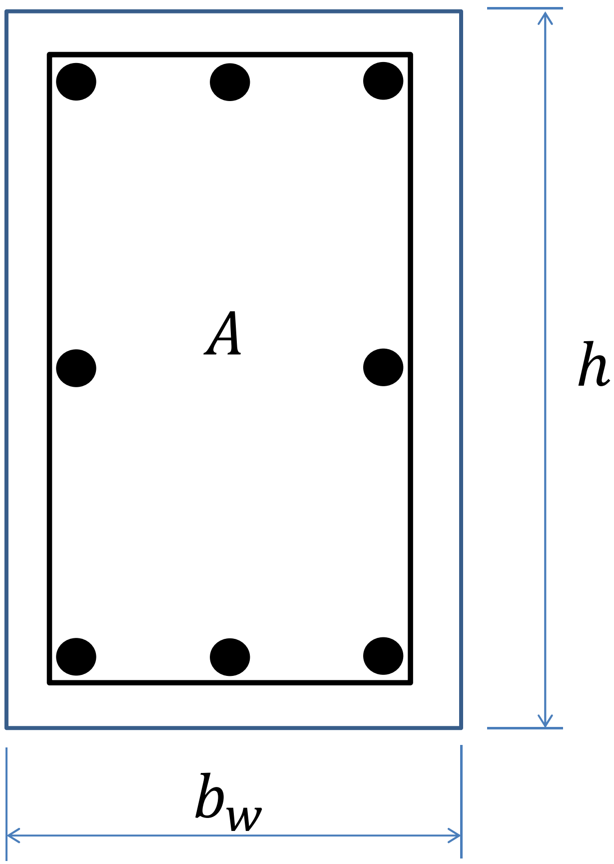

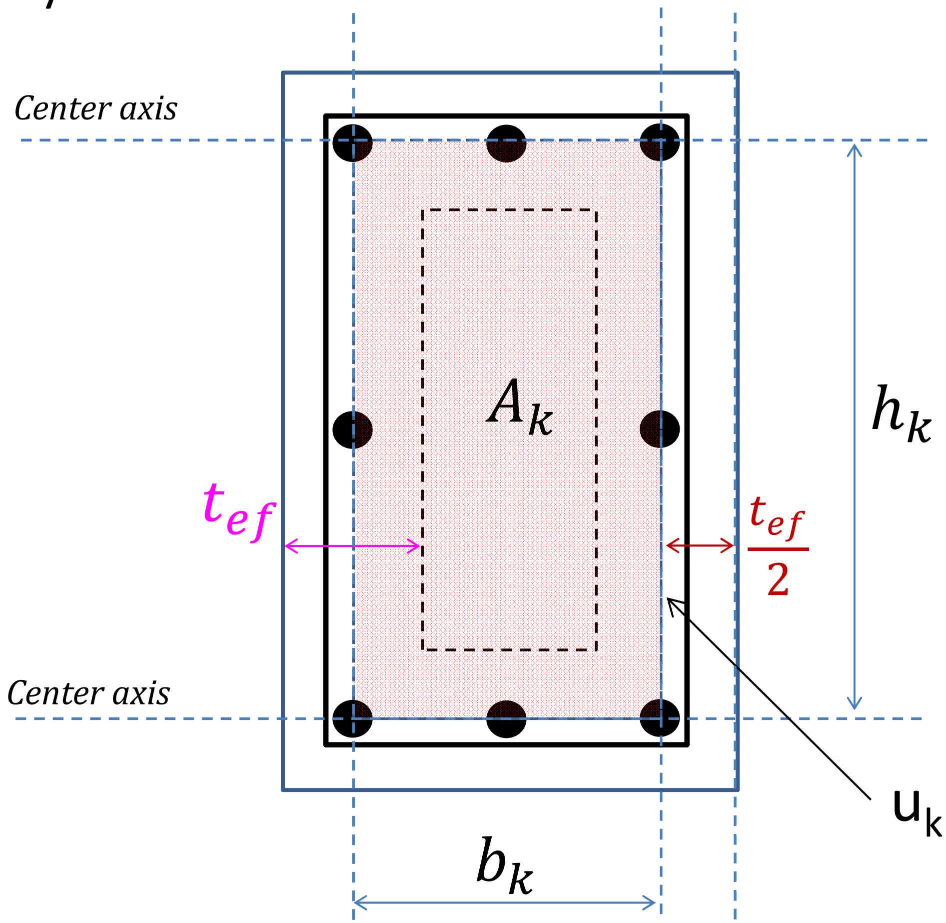

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Design model / Modelul de calcul

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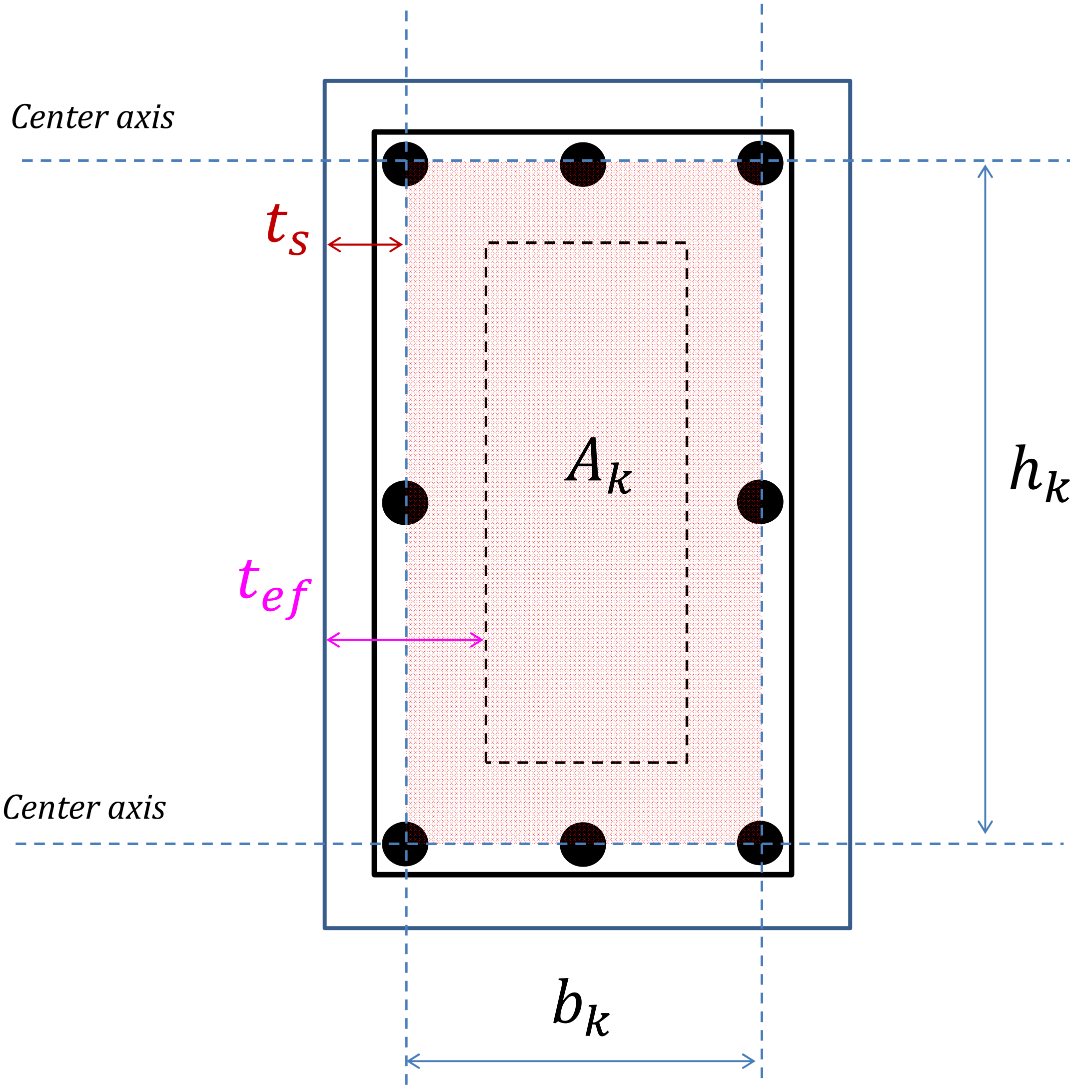
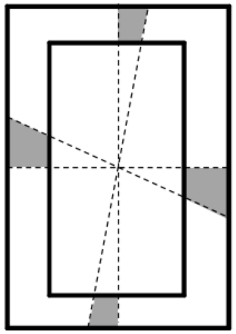


$$t_{ef} = \frac{A}{u} \geq t_{ef,min} = 2t_s$$

Why equivalent thin-walled sections?

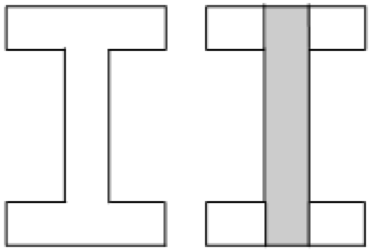
The greatest shear stresses are at edge of the cross section



Design model / Modelul de calcul

For complex shapes:

- may be divided into a series of sub-sections
- every sub-sections is modeled as an equivalent thin-walled section
- for non-solid sections the equivalent wall thickness should not exceed the actual wall thickness
- each sub-section may be designed separately
- the distribution of the acting torsional moment over the sub-sections should be in proportion to their uncracked torsional stiffnesses



$$T_{Ed,i} = \frac{I_{ti}}{\sum I_{ti}} T_{Ed}$$

$$I_t = \eta h b^3$$

h/b	1,0	1,2	1,4	1,6	1,8	2,0	2,2	2,4	2,6	2,8	3,0
η	0,140	0,163	0,185	0,203	0,216	0,229	0,232	0,235	0,239	0,248	0,246

Design model / Modelul de calcul

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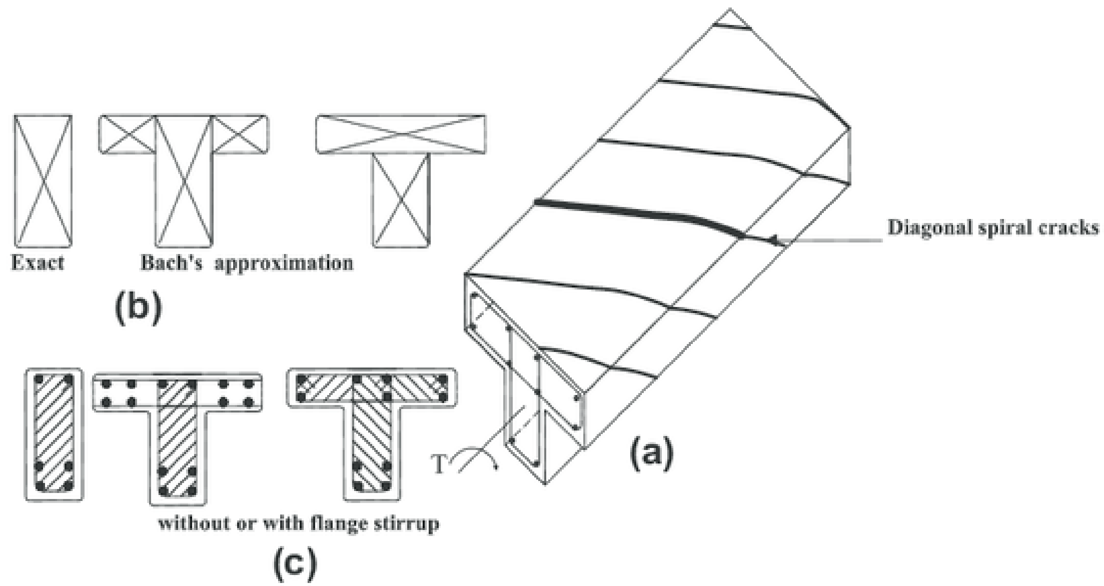
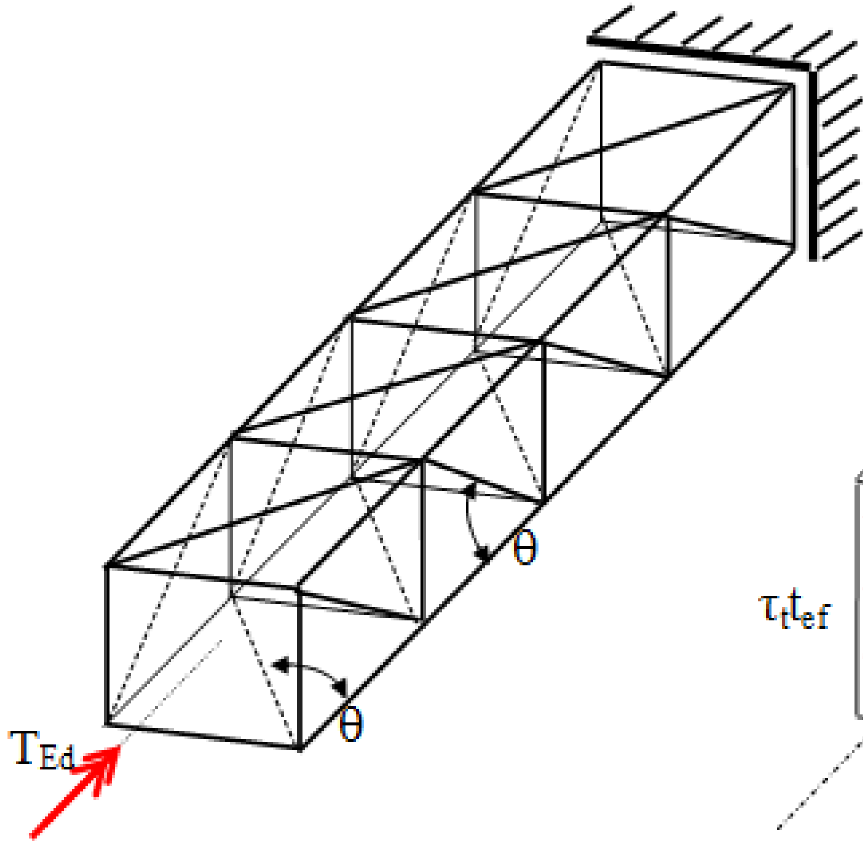
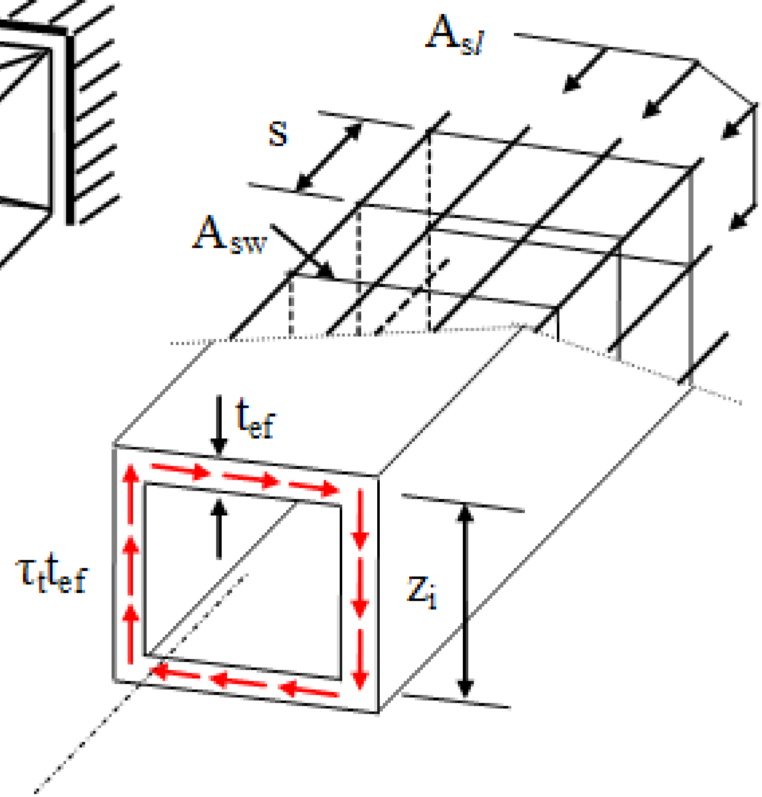


Design model / Modelul de calcul

Every wall includes a truss → 3D truss





Design model / Modelul de calcul

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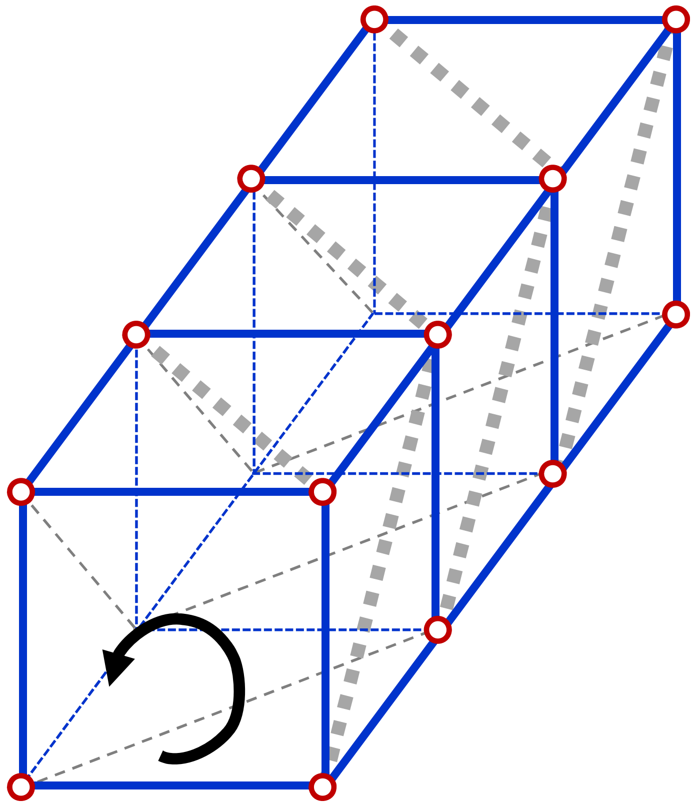

Design model / Modelul de calcul

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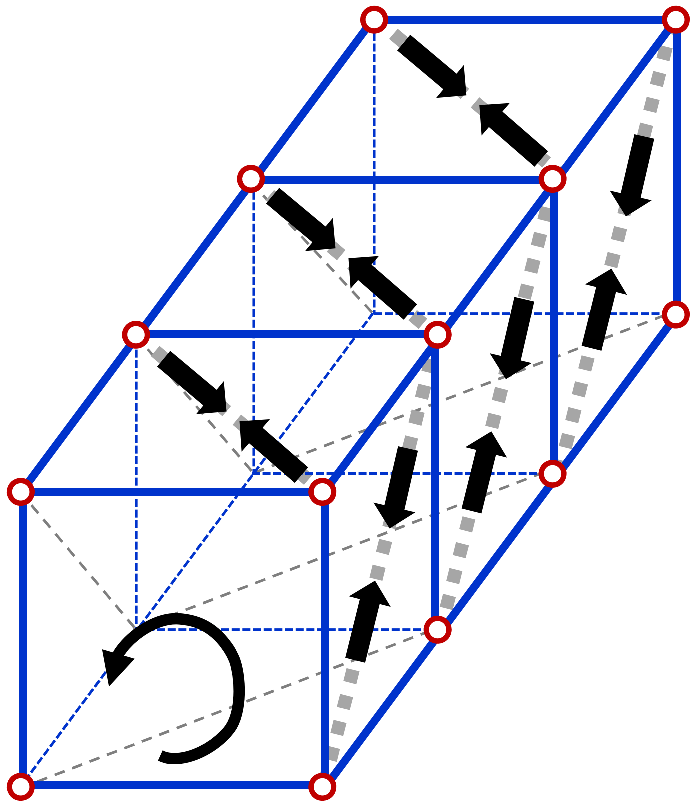
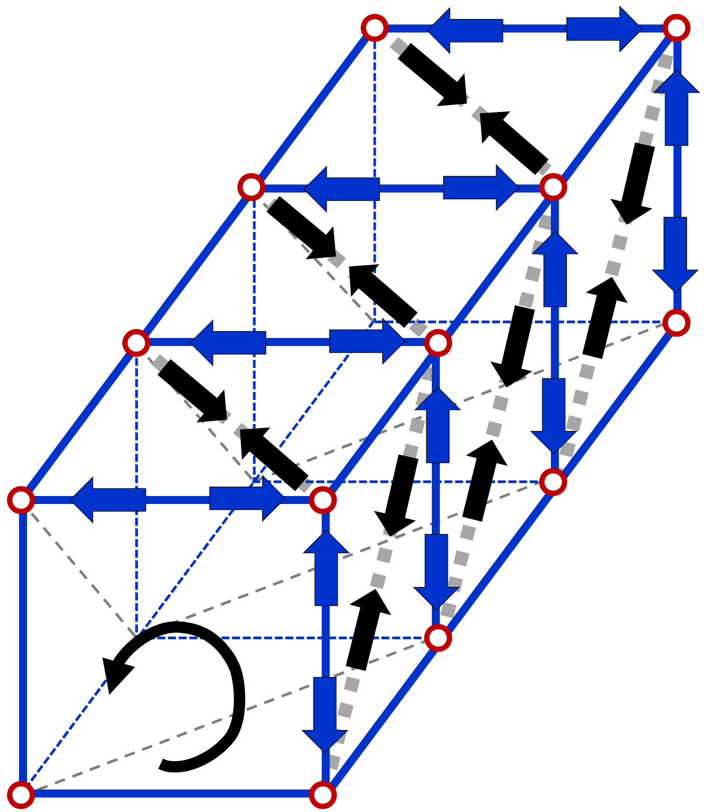


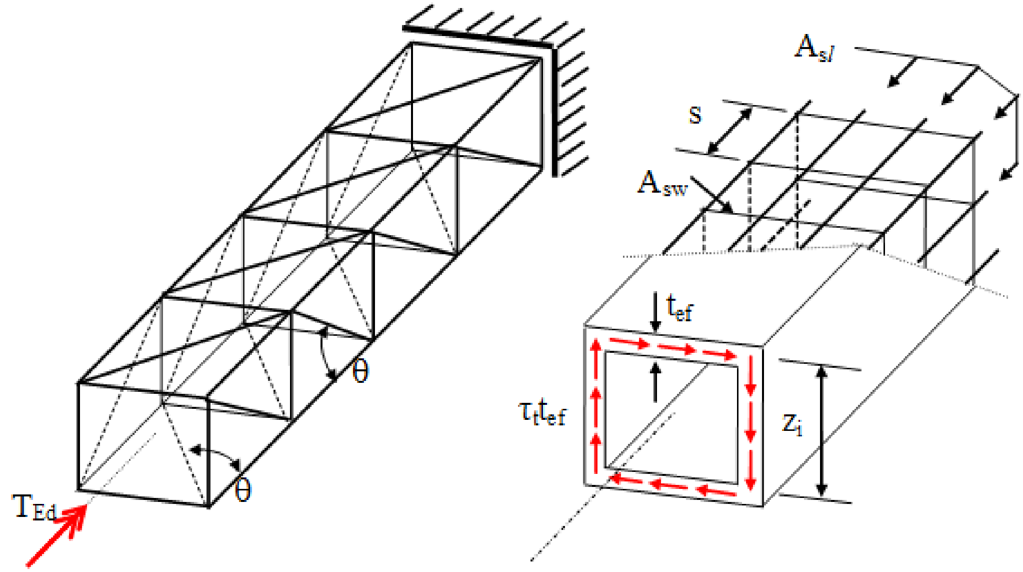
Design model / Modelul de calcul

Every wall includes a truss → 3D truss



Design model / Modelul de calcul

Every wall includes a truss → 3D truss



1) The effects of torsion and shear may be superimposed, assuming the same value for the strut inclination θ; **the same angle θ in every wall.**

2) The distribution of stirrups is constant along the element.

3) Longitudinal bars are distributed around the section; for calculation reasons longitudinal reinforcement is concentrated in the four corners.

4) T_{Ed} is replace by a flow of shear $\tau_t t_{ef}$

2.1 INTRODUCTION

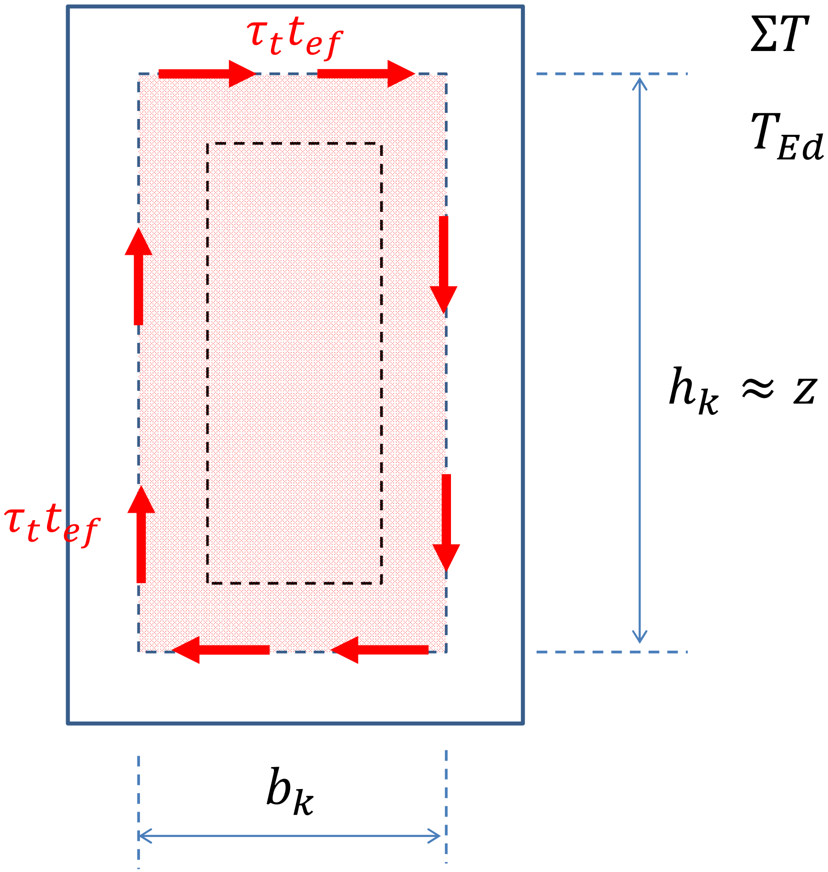
2.2 BEHAVIOR FOR TORSION

2.3 DESIGN MODEL

2.4 CALCULATION FOR TORSION

2.5 DETAILING OF REINFORCEMENT

Calculation for torsion / Calculul la torsiune



$$\Sigma T = 0$$

$$T_{Ed} = (\tau_t t_{ef}) \cdot h_k \cdot \frac{b_k}{2} \cdot 2 + (\tau_t t_{ef}) \cdot b_k \cdot \frac{h_k}{2} \cdot 2$$

shear force for wall "h_k" — shear force for wall "b_k"

$$T_{Ed} = 2(\tau_t t_{ef}) A_k$$

⇓

$$\tau_t t_{ef} = \frac{T_{Ed}}{2A_k}$$

→ shear force for a wall:

$$V_{Ed} = \tau_t t_{ef} z = \frac{T_{Ed}}{2A_k} z$$

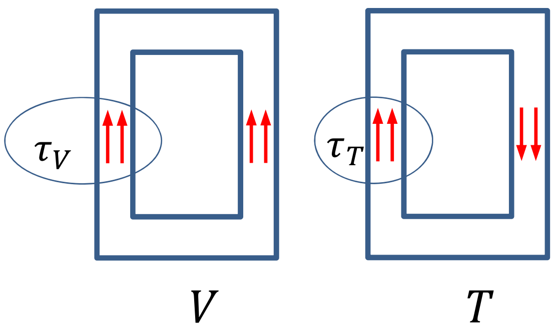
Calculation for torsion / Calculul la torsiune

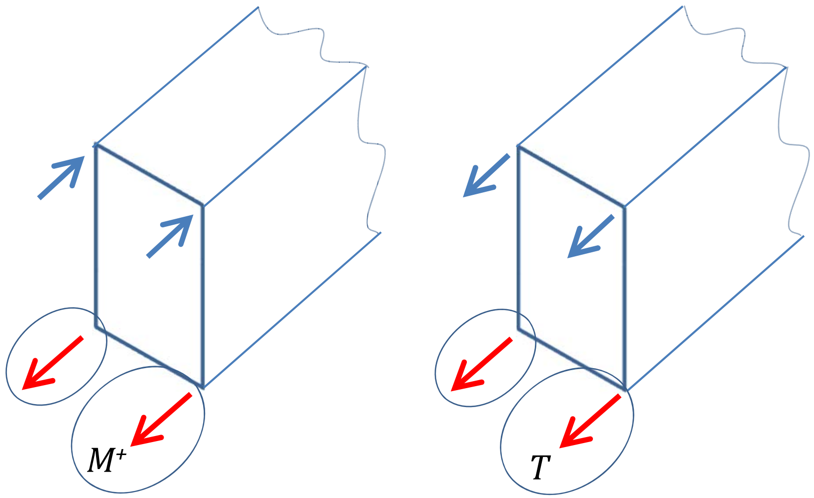
CAPACITY OF COMPRESSION STRUTS

Structural elements are subjected to $M_{Ed} + \boldsymbol{V_{Ed}} + \boldsymbol{T_{Ed}}$

→ shear stresses are induced

→ should take account superposition of the effects of torsion and shear





CAPACITY OF COMPRESSION STRUTS

FOR APPROXIMATELY RECTANGULAR SOLID SECTIONS

Calculation for combined shear and torsion is required ← **NO**

$$\frac{T_{Ed}}{T_{Rd,c}} + \frac{V_{Ed}}{V_{Rd,c}} \leq 1$$

YES → no reinforcement calculation required

Where

$T_{Rd,c} = 2A_k t_{ef} f_{ctd}$ - torsional cracking moment, with $\tau_t = f_{ctd}$

CAPACITY OF COMPRESSION STRUTS

THE MAXIMUM RESISTANCE OF A MEMBER SUBJECTED TO TORSION AND SHEAR IS LIMITED BY THE CAPACITY OF THE CONCRETE STRUTS

reconsider of the cross section ← **NO**

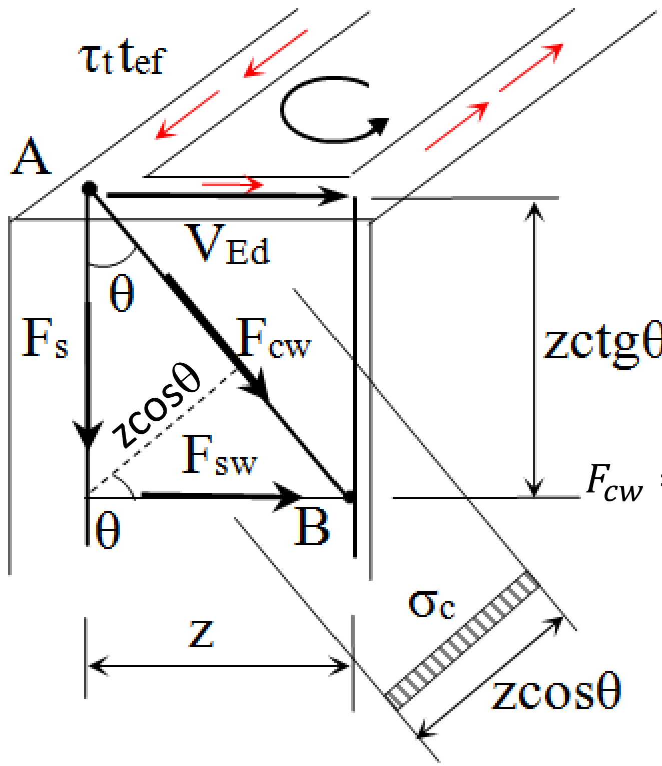
$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

YES → next step = reinforcement calculation

Notes about $V_{Rd,max}$
- in solid cross sections the full width of the web may be used
- for non-solid sections replace b_w by t_{ef}

CAPACITY OF COMPRESSED STRUTS

THE MAXIMUM RESISTANCE OF A MEMBER SUBJECTED TO TORSION AND SHEAR IS LIMITED BY THE CAPACITY OF THE CONCRETE STRUTS



$$F_{cw} = \sigma_c t_{ef} z \cos\theta$$

$$\Sigma F_A = 0$$

$$V_{Ed} = F_{cw} \sin\theta \quad \rightarrow$$

$$F_{cw} = V_{Ed}/\sin\theta$$

$$V_{Ed} = \tau_t t_{ef} z = \frac{T_{Ed}}{2A_k} z$$

$$\sigma_c t_{ef} z \cos\theta = \frac{T_{Ed}}{2A_k \sin\theta} z$$

$$\rightarrow \quad T_{Ed} = 2A_k \sigma_c t_{ef} \sin\theta \cos\theta$$

resisting torque by compression strut for $\sigma_c = f_{cd}$

EC2: $$T_{Rd,max} = 2\alpha_{cw} \nu f_{cd} A_k t_{ef} \sin\theta \cos\theta$$

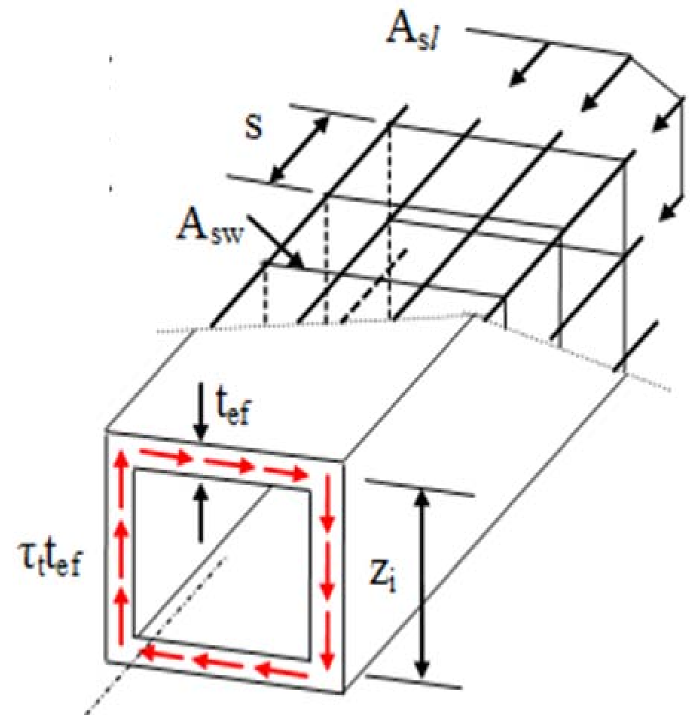
LONGITUDINAL REINFORCEMENT CALCULATION

$\frac{A_k}{u_k}$ - longitudinal reinforcement uniformly distributed on perimeter u_k

$\frac{A_{sl}}{u_k}z$ - longitudinal reinforcement for a wall

$\frac{A_{sl}}{u_k}z\sigma_s$ - corresponding tensile force in the wall

$$F_s = \frac{A_{sl}}{u_k}z\sigma_s$$



LONGITUDINAL REINFORCEMENT CALCULATION

$$\Sigma F_A = 0$$

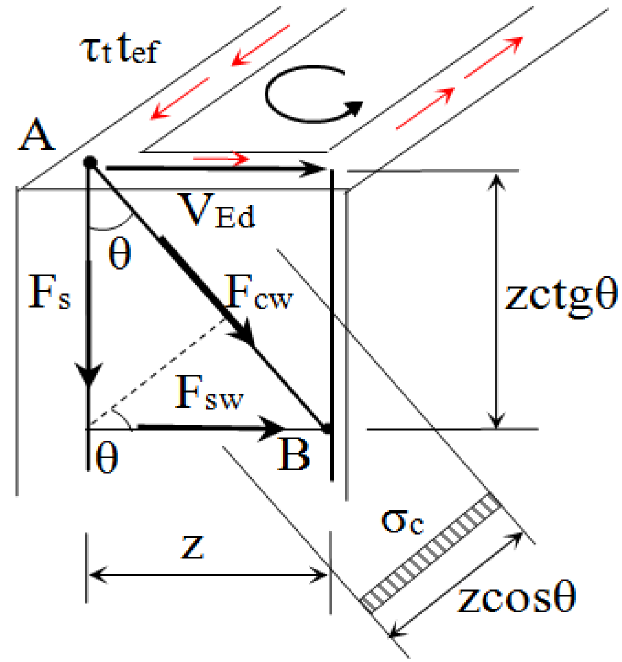
$$F_s = F_{cw} cos\theta$$

$$F_{cw} = V_{Ed}/sin\theta$$

$$F_s = V_{Ed}\ cot\theta$$

But $V_{Ed} = \frac{T_{Ed}}{2A_k} z$

→ $$F_s = \frac{T_{Ed}}{2A_k} z\ cot\theta$$



LONGITUDINAL REINFORCEMENT CALCULATION

$$F_s = \frac{A_{sl}}{u_k} z\sigma_s \quad \text{and} \quad F_s = \frac{T_{Ed}}{2A_k} z \cot\theta$$

$$\frac{A_{sl}}{u_k} z\sigma_s = \frac{T_{Ed}}{2A_k} z \cot\theta$$

$$T_{Ed} = \underbrace{2A_k \frac{A_{sl}}{u_k} \sigma_s \tan\theta}$$

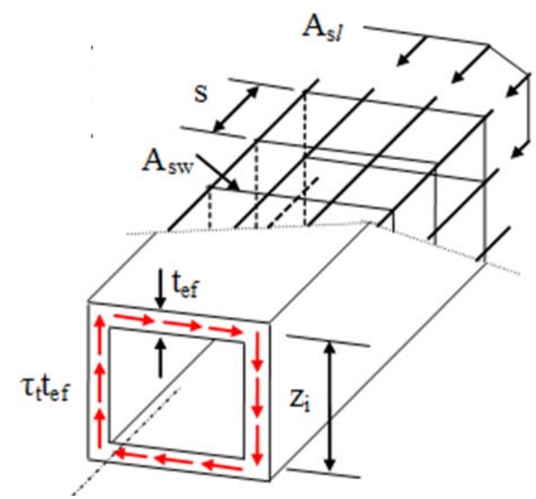
resisting torque by longitudinal bars for $\sigma_s = f_{yd}$

$$\rightarrow \quad T_{Rd,sl} = 2A_k \frac{A_{sl}}{u_k} f_{yd} \tan\theta$$

Required area of the longitudinal bars is obtained from $T_{Rd,sl} = T_{Ed}$

$$\rightarrow \quad \boxed{A_{sl} = \frac{T_{Ed} u_k}{2A_k f_{yd}} \cot\theta}$$

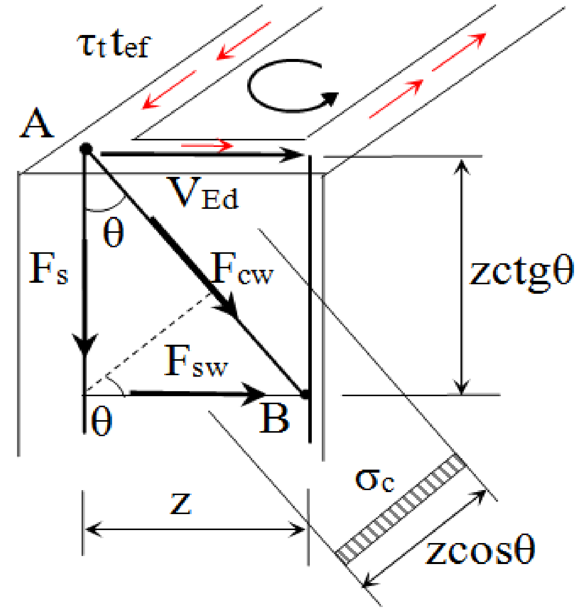
STIRRUP CALCULATION



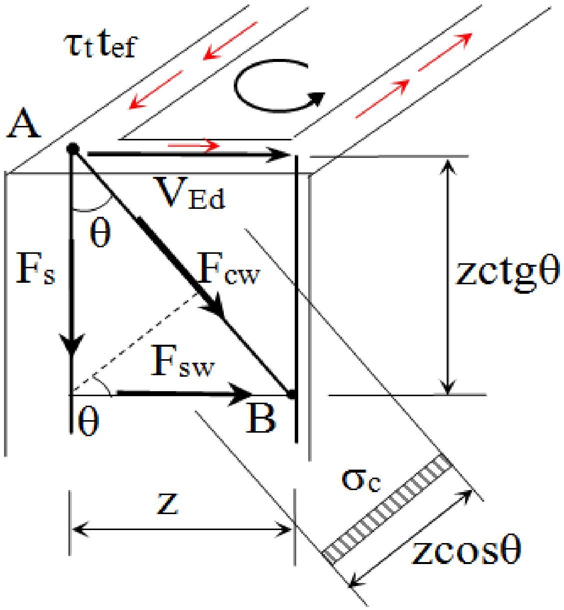
$\dfrac{A_{sw}}{s}$ - stirrup area distributed on unit length

$\dfrac{A_{sw}}{s} z\, cot\theta$ - area of all stirrups on length $z\, cot\theta$

$$\rightarrow \quad F_s = \frac{A_{sw}}{s} z \sigma_s\, cot\theta$$



STIRRUP CALCULATION



$$\Sigma F_B = 0$$

$$F_{sw} = F_{cw} sin\theta$$

$$V_{Ed} = F_{cw} sin\theta$$

$$\rightarrow \quad V_{Ed} = F_{sw}$$

$$\downarrow \quad \downarrow$$

$$\frac{T_{Ed}}{2A_k} z = \frac{A_{sw}}{s} z\sigma_s \, cot\theta$$

for $\sigma_s = f_{ywd}$ the resisting torque is obtained

$$T_{Rd,sw} = \frac{2A_{sw}A_k}{s} f_{ywd} \, cot\theta$$

Required stirrups are obtained from $T_{Rd,sw} = T_{Ed}$

$$\rightarrow \quad \left(\frac{A_{sw}}{s}\right)_{nec} = \frac{T_{Ed}}{2A_k f_{ywd}} tan\theta$$

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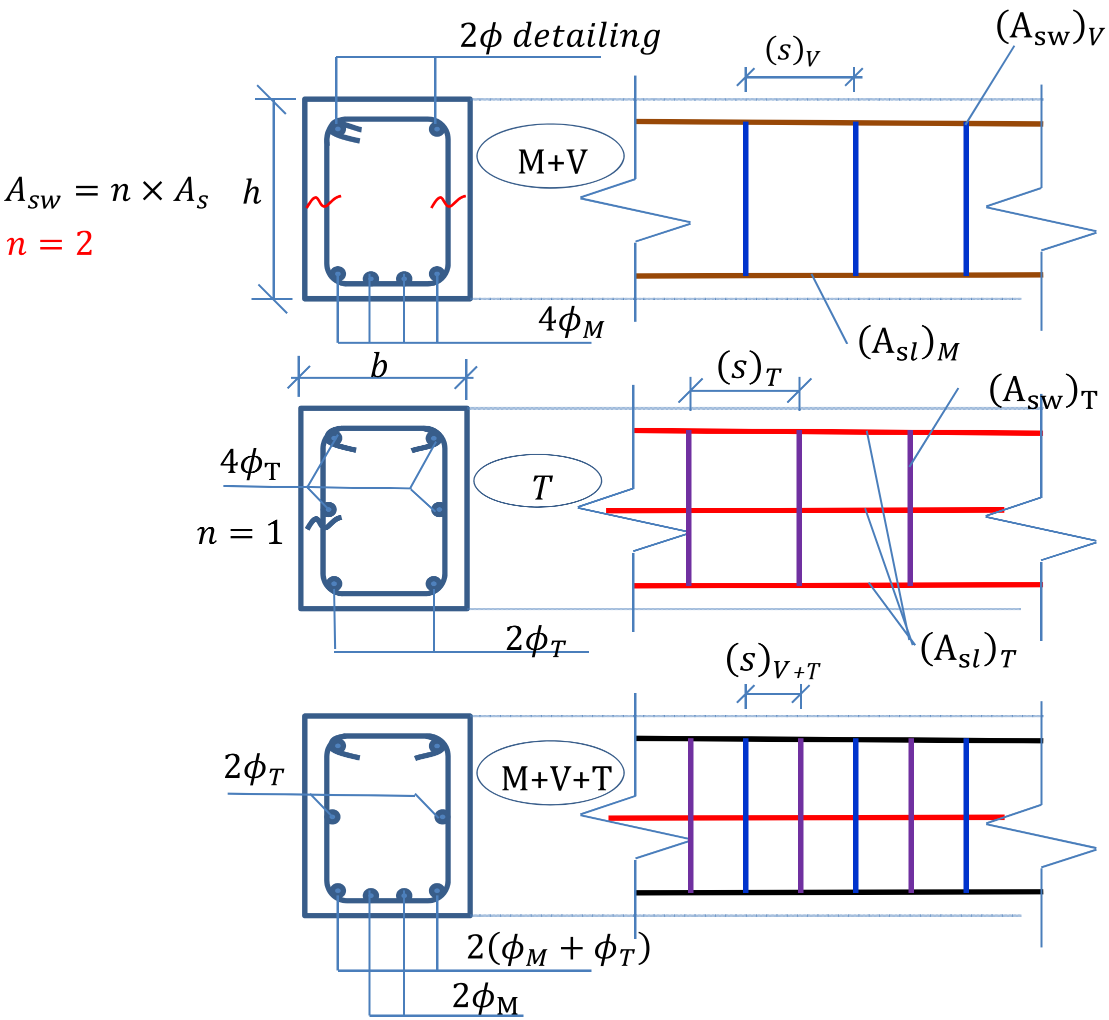
Detailing of reinforcement / Detalierea armăturilor

Structural elements are subjected to $M_{Ed} + V_{Ed} + T_{Ed}$

→ should take account of superposition of the effects of all the effects

M_{Ed}	V_{Ed}	T_{Ed}	Σ
A_s	-	A_{sl}	$A_s + A_{sl}$
-	$(A_{sw}/s)_V$	$(A_{sw}/s)_T$	$(A_{sw}/s)_{V+T}$

Detailing of reinforcement / Detalierea armăturilor



$$\left(\frac{A_{sw}}{s}\right)_V$$

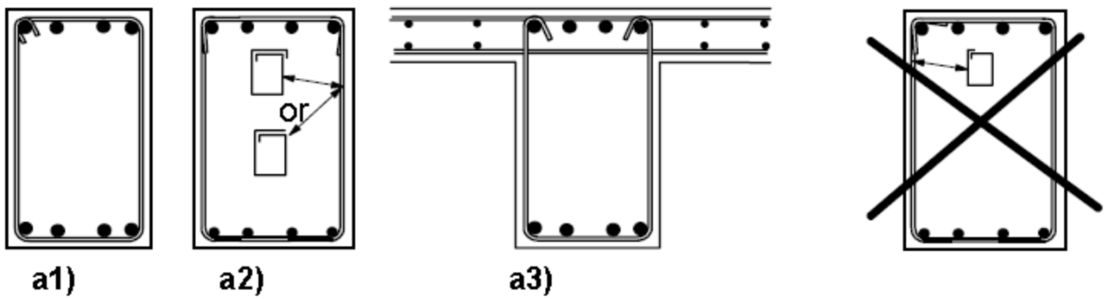
$$\left(\frac{A_{sw}}{s}\right)_T$$

$$\left(\frac{A_{sw}}{s}\right) = \left(\frac{A_{sw}}{s}\right)_V + \left(\frac{A_{sw}}{s}\right)_T$$

$$s = \frac{s_V \cdot s_T}{s_V + s_T}$$

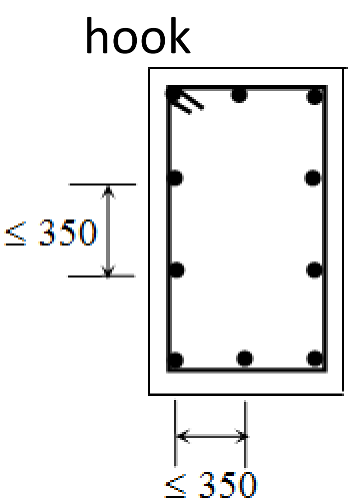
Detailing of reinforcement / Detalierea armăturilor

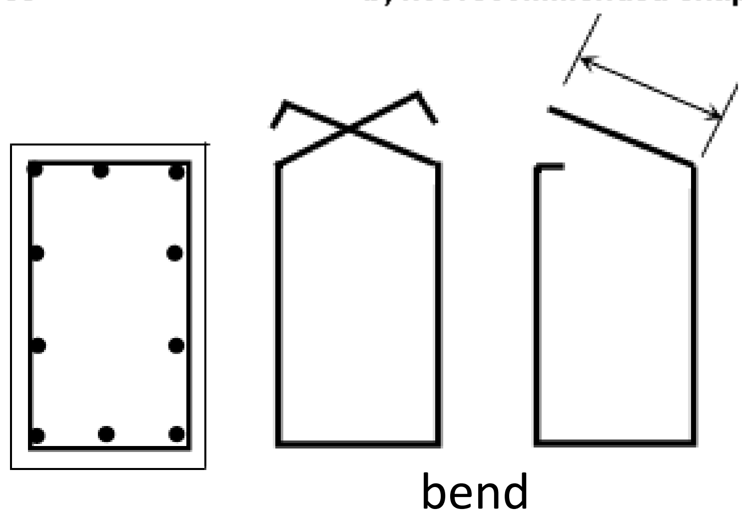
The torsion links should be closed and be anchored by means of laps or hooked ends and should form an angle of 90° with the axis of the structural element.

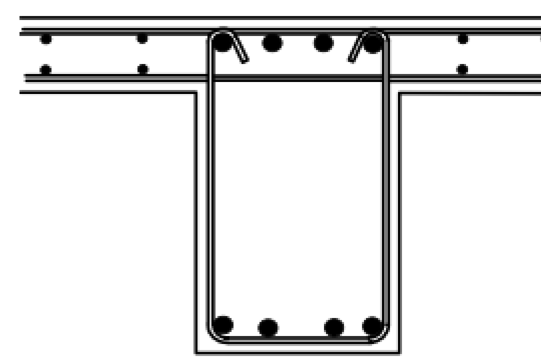


a) recommended shapes

b) not recommended shape

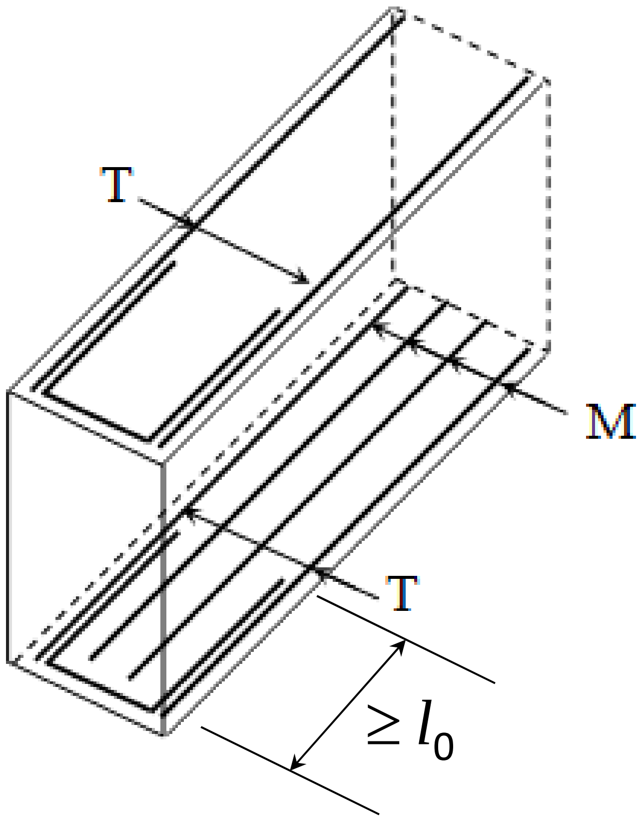




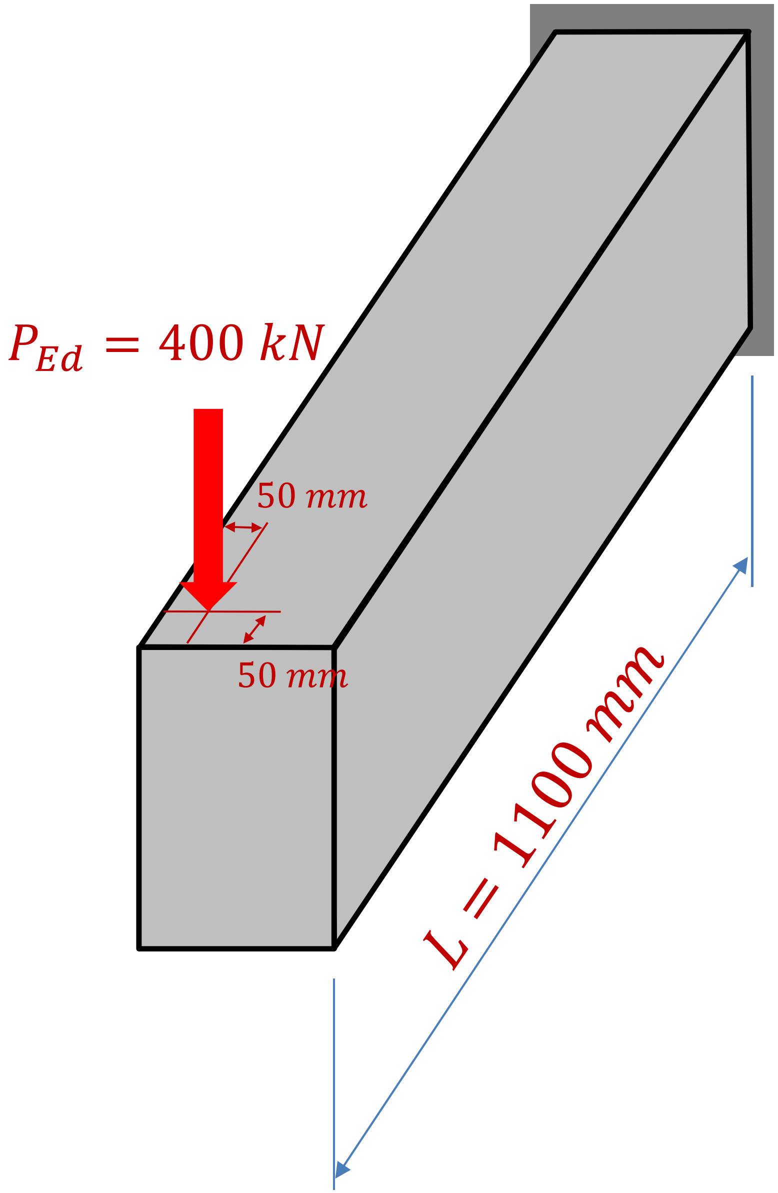


Detailing of reinforcement / Detalierea armăturilor

To anchor corner bars special shape reinforcements are used.



THANK YOU FOR YOUR ATTENTION!



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